PENGUIN SERIES

Accurate Mathematics

A Coursebook in Mathematics with Activities

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PETERSON PRESS
EDUCATION LEADS
A UNIT OF
NAMAN PUBLISHING (INDIA) PVT. LTD.

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Typesetting & Graphics by

Dreamshapers # 9897035745

Editors

Damini Kaushik

Yatin Kumar

Published by

PETERSON PRESS

EDUCATION LEADS

A UNIT OF NAMAN PUBLISHING (INDIA) PVT. LTD.

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Behind Silver Line School, Laxmipuram,

Rajpur Chungi, AGRA-282001

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Preface

Accurate Mathematics is an innovative series for classes 1 to 8 which is specially designed for the children of new generation. Children should enjoy learning Mathematics rather than be afraid of it. They should pose and solve meaningful problems with ease. The contents of the books are complete and carefully graded as per the novel approach to the teaching of 'Mathematics hands on experience', in perfect co-ordination with resources available in the learner's immediate environment. The series follows the 'explain, comprehend and practise essential drill application' approach. The chapters provide a clear understanding, emphasize an investigative and exploratory approach to teaching. Wherever necessary, theory is presented precisely in a style tailored to act as a tool for teachers and students.

The theory is presented in a very simple language and supported with examples from everyday life.

A large number of objective questions have been included, which will help students quickly test their knowledge and skill.

A separate chapter titled as 'Activities' has been included to connect maths with real-life situations.

Test papers will help learners practise and apply the concepts learnt.

Every effort has been made to keep the series error-free. However, constructive suggestions for the improvement of the next edition will be highly appreciated.

– Publisher

Contents

1 Rational Numbers

In the previous class, we have already learnt about natural numbers, whole numbers, integers and fractions. We have also studied about various operations on rational numbers. In this chapter, we will study about positive and negative rational numbers, comparison and the properties of operations on rational numbers.

RATIONAL NUMBERS

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called rational numbers.

Example. Each of the numbers $\frac{3}{8}, \frac{-5}{11}, \frac{8}{-17}$ and $\frac{-7}{-15}$ is a rational number.

Positive Rationals : *A rational number is said to be positive if its numerator and denominator are either both positive or both negative.*

Thus, $\frac{4}{5}$ and $\frac{-5}{-7}$ are both positive rationals.

Negative Rationals : *A rational number is said to be negative if its numerator and denominator are of opposite signs, i.e., one positive and the other negative.*

Thus, $\frac{-3}{7}$ and $\frac{2}{-11}$ are both negative rationals.

PROPERTIES OF RATIONAL NUMBERS

Property 1. *If $\frac{p}{q}$ is a rational number and m is a nonzero integer then $\frac{p}{q} = \frac{p \times m}{q \times m}$.*

Example. $\frac{-2}{5} = \frac{(-2)\times 2}{5\times 2} = \frac{(-2)\times 3}{5\times 3} = \frac{(-2)\times 4}{5\times 4} = \ldots$

$\Rightarrow \quad \frac{-2}{5} = \frac{-4}{10} = \frac{-6}{15} = \frac{-8}{20} = \ldots$

Such rational numbers are called **equivalent rational numbers**.

Property 2. *If $\frac{p}{q}$ is a* rational number and m is a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$.

Thus, we can write, $\frac{-21}{35} = \frac{-21 \div 7}{35 \div 7} = \frac{-3}{5}$.

Standard Form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in standard form if p and q are integers having no common divisor other than 1 and q is positive.

Example 1. *Express $\frac{36}{-60}$ in standard form.*

Solution: $\frac{36}{-60} = \frac{36 \times (-1)}{(-60) \times (-1)} = \frac{-36}{60}$

The greatest common divisor of 36 and 60 is 12.

$\therefore \quad \frac{-36}{60} = \frac{(-36) \div 12}{60 \div 12} = \frac{-3}{5}$

Hence, $\frac{-36}{60} = \frac{-3}{5}$ (in standard form).

Property 3. Let $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers. Then, $\frac{p}{q} = \frac{r}{s} \Leftrightarrow (p \times s) = (q \times r)$

$$\frac{p}{q} \times \frac{r}{s}$$

$p \times s = q \times r$

Comparison of Rational Numbers

It is clear that:

(i) every positive rational number is greater than 0,

(ii) every negative rational number is less than 0.

GENERAL METHOD OF COMPARING RATIONAL NUMBERS

Step 1: Express each rational numbers with positive denominator.

Step 2: Take the LCM of denominators of all rational numbers.

Step 3: Express each rational number (obtained in Step 1) with this LCM as the common denominator.

Step 4: The number having the greater numerator is greater.

Example 2. *Which of the numbers $\frac{3}{-5}$ or $\frac{-4}{7}$ is greater?*

Solution: First we write each of the given numbers with positive denominator.

One number $= \frac{3}{-5} = \frac{3 \times (-1)}{(-5) \times (-1)} = \frac{-3}{5}$

The other number $= \frac{-4}{7}$

LCM of 5 and 7 = 35.

$\therefore \quad \frac{-3}{5} = \frac{(-3) \times 7}{5 \times 7} = \frac{-21}{35}$ and $\frac{-4}{7} = \frac{(-4) \times 5}{7 \times 5} = \frac{-20}{35}$

Clearly, $-21 < -20 \quad \therefore \frac{-21}{35} < \frac{-20}{35}$

Hence, $\frac{-3}{5} < \frac{-4}{7}$ *i.e.,* $\frac{3}{-5} < \frac{-4}{7}$

Example 3. *Arrange the numbers $\frac{-3}{5}, \frac{7}{-10}$ and $\frac{-5}{8}$ in ascending order.*

Solution: First we write each of the given numbers with positive denominator. We have:

$$\frac{7}{-10} = \frac{7 \times (-1)}{(-10) \times (-1)} = \frac{-7}{10}$$

Thus, the given numbers are $\frac{-3}{5}, \frac{-7}{10}$ and $\frac{-5}{8}$.

LCM of 5, 10 and 8 is 40.

Now, $\frac{-3}{5} = \frac{(-3)\times 8}{5\times 8} = \frac{-24}{40}; \frac{-7}{10} = \frac{(-7)\times 4}{10\times 4} = \frac{-28}{40}$ and $\frac{-5}{8} = \frac{(-5)\times 5}{8\times 5} = \frac{-25}{40}$.

Clearly, $\frac{-28}{40} < \frac{-25}{40} < \frac{-24}{40}$.

Hence, $\frac{-7}{10} < \frac{-5}{8} < \frac{-3}{5}$ *i.e.*, $\frac{7}{-10} < \frac{-5}{8} < \frac{-3}{5}$.

Exercise 1A

1. Express $\frac{-3}{5}$ as a rational number with denominator:

(a) 15 (b) −25 (c) 30 (d) −45

2. Express $\frac{-48}{60}$ as a rational number with denominator 5.

3. Express $\frac{-42}{98}$ as a rational number with denominator 7.

4. Express $\frac{24}{-30}$ as a rational number with numerator 4.

5. Express each of the following rational numbers in standard form:

(a) $\frac{-14}{35}$ (b) $\frac{-24}{48}$ (c) $\frac{27}{-63}$ (d) $\frac{-45}{-72}$

6. Which of the two rational numbers is greater in the given pair?

(a) $\frac{-12}{5}$ or -3 (b) $\frac{4}{-5}$ or $\frac{-7}{10}$ (c) $\frac{9}{-13}$ or $\frac{7}{-12}$ (d) $\frac{-1}{3}$ or $\frac{4}{-5}$

(e) $\frac{7}{-9}$ or $\frac{-5}{8}$ (f) $\frac{-4}{3}$ or $\frac{-8}{7}$

7. Fill in the boxes with the correct symbol out of >, = and <:

(a) $-2 \square \frac{-13}{5}$ (b) $0 \square \frac{-3}{-5}$ (c) $\frac{-2}{3} \square \frac{5}{-8}$ (d) $\frac{-8}{9} \square \frac{-9}{10}$

(e) $\frac{-3}{7} \square \frac{6}{-13}$ (f) $\frac{5}{-13} \square \frac{-35}{91}$

8. Arrange the following rational numbers in ascending order:

(a) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$ (b) $\frac{4}{-9}, \frac{-5}{12}, \frac{7}{-18}, \frac{-2}{3}$

(c) $\frac{-4}{7}, \frac{-9}{14}, \frac{13}{-28}, \frac{-23}{42}$ (d) $\frac{3}{-5}, \frac{-7}{10}, \frac{-11}{15}, \frac{-13}{20}$

9. Arrange the following rational numbers in descending order:

(a) $\frac{-5}{6}, \frac{-7}{12}, \frac{-13}{18}, \frac{23}{-24}$ (b) $\frac{-3}{10}, \frac{7}{-15}, \frac{-11}{20}, \frac{17}{-30}$

(c) $\frac{-10}{11}, \frac{-19}{22}, \frac{-23}{33}, \frac{-39}{44}$ (d) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$

10. Which of the following statements are true and which are false?

(a) 0 is a whole number but it is not a rational number.

(b) Every whole number is a rational number.

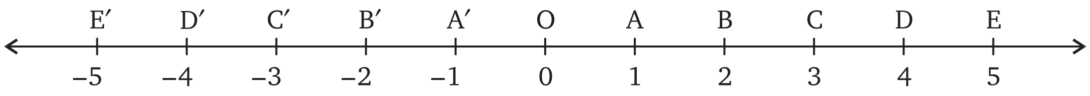
(c) Every integer is a rational number.

REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

In the previous class we have learnt how to represent integers on the number line.

Let us review it.

Draw any line. Take a point O on it. Call it 0 (zero). Set off equal distances on the right as well as on the left of O. Such a distance is known as a unit length. Clearly, the points A, B, C, D and E represent the integers 1, 2, 3, 4 and 5 respectively and the points A', B', C', D' and E' represent the integers $-1, -2, -3, -4$ and -5 respectively.



Thus, we may represent any integer by a point on the number line. Clearly, every positive integer lies to the right of O and every negative integer lies to the left of O.

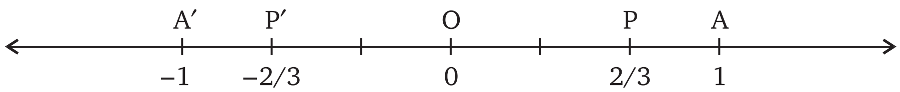
Similarly we can represent rational numbers.

Consider the following examples.

Example 1. *Represent* $\frac{2}{3}$ *and* $-\frac{2}{3}$ *on the number line.*

Solution: Draw a line. Take a point O on it. Let it represent 0. From O set off unit distances OA and OA' to the right and left of O respectively.

Divide OA into 3 equal parts. Let OP be the segment showing 2 parts out of 3. Then, the point P represents the rational number $\frac{2}{3}$.



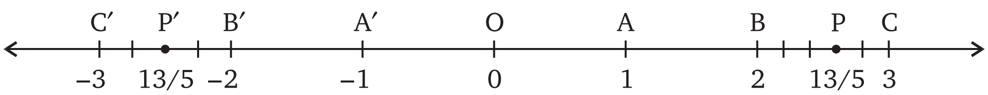
Again divide OA' into 3 equal parts. Let OP' be the segment consisting of 2 parts out of these 3 parts. Then, the point P' represents the rational number $-\frac{2}{3}$.

Example 2. *Represent* $\frac{13}{5}$ *and* $\frac{-13}{5}$ *on the number line.*

Solution: Draw a line. Take a point O on it. Let it represent 0.

Now, $\frac{13}{5} = 2\frac{3}{5} = 2 + \frac{3}{5}$.

From O, set off unit distances OA, AB and BC to the right of O. Clearly, the points A, B and C represent the integers 1, 2 and 3 respectively. Now, take 2 units OA and AB, and divide the third unit BC into 5 equal parts. Take 3 parts out of these 5 parts to reach at a point P. Then, the point P represents the rational number $\frac{13}{5}$.



Again, from O, set off unit distances to the left. Let these segments be OA', $A'B'$, $B'C'$, etc. Then, clearly the points A', B' and C' represent the integers $-1, -2, -3$ respectively.

Now, $\frac{-13}{5} = -\left(2 + \frac{3}{5}\right)$.

Take 2 full unit lengths to the left of O. Divide the third unit $B'C'$ into 5 equal parts. Take 3 parts out of these 5 parts to reach a point P'.

Then, the point P' represents the rational number $\frac{-13}{5}$.

Thus, we can represent every rational number by a point on the number line.

Exercise 1B

1. Represent each of the following numbers on the number line:

(a) $\frac{2}{7}$ (b) $\frac{1}{3}$ (c) $2\frac{2}{5}$ (d) $4\frac{2}{3}$

2. Represent each of the following numbers on the number line:

(a) $\frac{-3}{4}$ (b) $\frac{-1}{3}$ (c) $-3\frac{1}{7}$ (d) $-4\frac{3}{5}$

3. Which of the following statements are true and which are false?

(a) The rational number $\frac{-18}{-13}$ lies to the left of 0 on the number line.

(b) The rational numbers $\frac{1}{3}$ and $\frac{-5}{2}$ are on opposite sides of 0 on the number line.

(c) $\frac{-12}{7}$ lies to the right of 0 on the number line.

(d) $\frac{-3}{5}$ lies to the left of 0 on the number line.

ADDITION OF RATIONAL NUMBERS

Case 1. When the denominators of given rational numbers are same.

Let $\frac{p}{q}$ and $\frac{r}{q}$ be two rational numbers, then $\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$.

Example 1. *Find the sum:*

(a) $\frac{8}{17} + \frac{-5}{17}$ (b) $\frac{19}{-33} + \frac{-8}{33}$

Solution: (a) $\frac{8}{17} + \frac{-5}{17} = \frac{8+(-5)}{17} = \frac{8-5}{17} = \frac{3}{17}$

(b) $\frac{19}{-33} + \frac{-8}{33} = \frac{-19}{33} + \frac{-8}{33} = \frac{(-19)+(-8)}{33} = \frac{-19-8}{33} = \frac{-27}{33} = \frac{-9}{11}$

Case 2. When the denominators of given rational numbers are different.

METHOD :

1. Convert all unlike rational numbers to like rational numbers by taking LCM of denominators.
2. Express each of the given numbers with the LCM as their common denominator.

3. Now simply add.

Let $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers, then $\frac{p}{q}+\frac{r}{s}=\frac{ps+rq}{qs}$.

Example 2. *Find the sum:* $\frac{-5}{6}+\frac{3}{4}$

Solution: The denominator of the given rational numbers are 6 and 4 respectively.

LCM of 6 and 4 $=(2\times2\times3)=12$

2	6, 4
2	3, 2
3	3, 1
	1, 1

Now, $\frac{-5}{6}=\frac{(-5)\times2}{6\times2}=\frac{-10}{12}$

and $\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}$

$\therefore \left(\frac{-5}{6}+\frac{3}{4}\right)=\left(\frac{-10}{12}+\frac{9}{12}\right)=\frac{(-10)+9}{12}=\frac{-1}{12}$

Short-Cut Method

Example 3. *Find the sum:* $\frac{-9}{16}+\frac{5}{12}$.

4	16, 12
	4, 3

Solution: LCM of 16 and 12 $=(4\times4\times3)=48$.

$\therefore \frac{-9}{16}+\frac{5}{12}=\frac{3\times(-9)+4\times5}{48}=\frac{(-27)+20}{48}=\frac{-7}{48}$.

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Property 1 (Closure Property): *The sum of two rational numbers is always a rational number.*

If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers, then $\left(\frac{p}{q}+\frac{r}{s}\right)$ is also a rational number.

Examples. (a) Consider the rational numbers $\frac{1}{3}$ and $\frac{3}{4}$. Then,

$$\left(\frac{1}{3}+\frac{3}{4}\right)=\frac{(4+9)}{12}=\frac{13}{12}, \text{ which is a rational number.}$$

(b) Consider the rational numbers $\frac{-2}{3}$ and $\frac{4}{5}$. Then,

$$\left(\frac{-2}{3}+\frac{4}{5}\right)=\frac{(-10+12)}{15}=\frac{2}{15}, \text{ which is a rational number.}$$

(c) Consider the rational numbers $\frac{-5}{12}$ and $\frac{-1}{4}$. Then,

$$\left(\frac{-5}{12}+\frac{-1}{4}\right)=\frac{\{-5+(-3)\}}{12}=\frac{-8}{12}=\frac{-2}{3}, \text{ which is a rational number.}$$

Property 2 (Commutative Law): *Two rational numbers can be added in any order.*

Thus for any two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, we have

$$\left(\frac{p}{q}+\frac{r}{s}\right)=\left(\frac{r}{s}+\frac{p}{q}\right)$$

Examples. (a) $\left(\frac{1}{2}+\frac{3}{4}\right)=\frac{(2+3)}{4}=\frac{5}{4}$ and $\left(\frac{3}{4}+\frac{1}{2}\right)=\frac{(3+2)}{4}=\frac{5}{4}$.

$$\therefore \quad \left(\frac{1}{2}+\frac{3}{4}\right)=\left(\frac{3}{4}+\frac{1}{2}\right)$$

(b) $\left\{\frac{3}{8}+\frac{-5}{6}\right\}=\frac{\{9+(-20)\}}{24}=\frac{-11}{4}$ and $\left\{\frac{-5}{6}+\frac{3}{8}\right\}=\frac{\{-20+9\}}{24}=\frac{-11}{24}$.

$$\therefore \quad \left(\frac{3}{8}+\frac{-5}{6}\right)=\left(\frac{-5}{6}+\frac{3}{8}\right)$$

(c) $\left(\frac{-1}{2}+\frac{-2}{3}\right)=\frac{\{(-3)+(-4)\}}{6}=\frac{-7}{6}$ and $\left(\frac{-2}{3}+\frac{-1}{2}\right)=\frac{\{(-4)+(-3)\}}{6}=\frac{-7}{6}$.

$$\therefore \quad \left(\frac{-1}{2}+\frac{-2}{3}\right)=\left(\frac{-2}{3}+\frac{-1}{2}\right)$$

Property 3 (Associative Law): *While adding three rational numbers, they can be grouped in any order.*

Thus, for any three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{m}{n}$, we have

$$\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{m}{n}=\frac{p}{q}+\left(\frac{r}{s}+\frac{m}{n}\right)$$

Example. Consider three rationals $\frac{-2}{3}$, $\frac{5}{7}$ and $\frac{1}{6}$. Then,

$$\left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{-14+15}{21}+\frac{1}{6}\right\}=\left(\frac{1}{21}+\frac{1}{6}\right)=\frac{(2+7)}{42}=\frac{9}{42}=\frac{3}{14}$$

and $$\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}=\left\{\frac{-2}{3}+\frac{(30+7)}{42}\right\}$$

$$=\left(\frac{-2}{3}+\frac{37}{42}\right)=\frac{(-28+37)}{42}=\frac{9}{42}=\frac{3}{14}$$

$$\therefore \quad \left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}$$

Property 4 (Existence of Additive Identity): 0 *is a rational number such that the sum of any rational number and* 0 *is the rational number itself.*

Thus, $\left(\frac{p}{q}+0\right)=\left(0+\frac{p}{q}\right)=\frac{p}{q}$, for every rational number $\frac{p}{q}$.

0 is called the ***additive identity*** for rationals.

Examples. (a) $\left(\frac{3}{5}+0\right)=\left(\frac{3}{5}+\frac{0}{5}\right)=\frac{(3+0)}{5}=\frac{3}{5}$ and similarly, $\left(0+\frac{3}{5}\right)=\frac{3}{5}$.

$$\therefore \quad \left(\frac{3}{5}+0\right)=\left(0+\frac{3}{5}\right)=\frac{3}{5}$$

(b) $\left(\frac{-2}{3}+0\right)=\left(\frac{-2}{3}+\frac{0}{3}\right)=\frac{(-2+0)}{3}=\frac{-2}{3}$ and similarly, $\left(0+\frac{-2}{3}\right)=\frac{-2}{3}$.

$$\therefore \quad \left(\frac{-2}{3}+0\right)=\left(0+\frac{-2}{3}\right)=\frac{-2}{3}$$

Property 5 (Existence of Additive Inverse): *For every rational number* $\frac{p}{q}$*, there exists a rational number* $\frac{-p}{q}$ *such that* $\left(\frac{p}{q}+\frac{-p}{q}\right)=\frac{\{p+(-p)\}}{q}=\frac{0}{q}=0$ *and similarly,* $\left(\frac{-p}{q}+\frac{p}{q}\right)=0.$

Thus, $$\left(\frac{p}{q}+\frac{-p}{q}\right)=\left(\frac{-p}{q}+\frac{p}{q}\right)=0$$

$\frac{-p}{q}$ is called the ***additive inverse*** of $\frac{p}{q}$.

Example. $\left(\frac{4}{7}+\frac{-4}{7}\right)=\frac{\{4+(-4)\}}{7}=\frac{0}{7}=0$ and similarly, $\left(\frac{-4}{7}+\frac{4}{7}\right)=0$

$\therefore$ $$\left(\frac{4}{7}+\frac{-4}{7}\right)=\left(\frac{-4}{7}+\frac{4}{7}\right)=0.$$

Thus, $\frac{4}{7}$ and $\frac{-4}{7}$ are additive inverses of each other.

SUBTRACTION OF RATIONAL NUMBERS

For rational number $\frac{p}{q}$ *and* $\frac{r}{s}$*, we define:*

$$\left(\frac{p}{q}-\frac{r}{s}\right)=\frac{p}{q}+\left(\frac{-r}{s}\right)=\frac{p}{q}+\left(\textbf{additive inverse of } \frac{r}{s}\right)$$

Solved Examples

Example 1. *Find the additive inverse of:*

(*a*) $\frac{-6}{-7}$ (*b*) $\frac{9}{-11}$ (*c*) $\frac{-15}{8}$ (*d*) $\frac{5}{9}$

Solution: (a) We may write, $\frac{-6}{-7}=\frac{(-6)\times(-1)}{(-7)\times(-1)}=\frac{6}{7}$

Hence, its additive inverse is $\frac{-6}{7}$.

(b) In standard form, we write $\frac{9}{-11}$ as $\frac{-9}{11}$.

Hence, its additive inverse is $\frac{9}{11}$.

(c) Additive inverse of $\frac{-15}{8}$ is $\frac{15}{8}$.

(d) Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

Example 2. (*a*) *Subtract* $\frac{-5}{7}$ *from* $\frac{-2}{5}$. (*b*) *Subtract* $\frac{3}{4}$ *from* $\frac{2}{3}$.

Solution: (a) $\left\{\frac{-2}{5}-\left(\frac{-5}{7}\right)\right\}=\frac{-2}{5}+\left(\text{additive inverse of } \frac{-5}{7}\right).$

$$=\left(\frac{-2}{5}+\frac{5}{7}\right) \qquad \left[\because \text{ additive inverse of } \frac{-5}{7} \text{ is } \frac{5}{7}\right]$$

$$=\frac{(-14+25)}{35}=\frac{11}{35}$$

(b) $\left(\frac{2}{3}-\frac{3}{4}\right)=\frac{2}{3}+\left(\text{additive inverse of } \frac{3}{4}\right)$

$$=\left(\frac{2}{3}+\frac{-3}{4}\right)=\frac{\{8+(-9)\}}{12}=\frac{-1}{12}.$$

Example 3. What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?

Solution: Let the required number to be added be x. Then,

$$\frac{-7}{8}+x=\frac{4}{9} \quad \Rightarrow \quad x=\frac{4}{9}+\left(\text{additive inverse of } \frac{-7}{8}\right)$$

$$\Rightarrow \quad x=\left(\frac{4}{9}+\frac{7}{8}\right)=\frac{(32+63)}{72}=\frac{95}{72}$$

$$\Rightarrow \quad x=\frac{95}{72}$$

Hence, the required number is $\frac{95}{72}$.

Example 4. *The sum of two rational numbers is –5. If one of them is* $\frac{-13}{6}$*, find the other.*

Solution: Let the other number be x. Then,

$$x+\left(\frac{-13}{6}\right)=-5 \quad \Rightarrow \quad x=-5+\left(\text{additive inverse of } \frac{-13}{6}\right)$$

$$\Rightarrow \quad x=\left(-5+\frac{13}{6}\right)=\left(\frac{-5}{1}+\frac{13}{6}\right)=\frac{(-30+13)}{6}$$

$$\Rightarrow \quad x=\frac{-17}{6}$$

Hence, the required number is $\frac{-17}{6}$.

Example 5. *Simplify:* $\left(\frac{4}{7}+\frac{-8}{9}+\frac{-5}{21}+\frac{1}{3}\right).$

Solution: Using the rearrangement property, we have:

$$\frac{4}{7}+\frac{-8}{9}+\frac{-5}{21}+\frac{1}{3}=\left(\frac{4}{7}+\frac{-5}{21}\right)+\left(\frac{-8}{9}+\frac{1}{3}\right)$$

$$=\frac{\{12+(-5)\}}{21}+\frac{\{-8+3\}}{9}$$

$$=\left(\frac{7}{21}+\frac{-5}{9}\right)=\frac{\{21+(-35)\}}{63}=\frac{-14}{63}=\frac{-2}{9}$$

Example 6. *Evaluate* $\frac{3}{5}+\frac{7}{3}+\frac{-11}{5}+\frac{-2}{3}$.

Solution: Using the commutative and associative laws, it follows that we may arrange the terms in any manner suitably. Using this rearrangement property, we have:

$$\frac{3}{5}+\frac{7}{3}+\frac{-11}{5}+\frac{-2}{3}=\left(\frac{3}{5}+\frac{-11}{5}\right)+\left(\frac{7}{3}+\frac{-2}{3}\right)$$
$$=\frac{\{3+(-11)\}}{5}+\frac{\{7+(-2)\}}{3}=\frac{-8}{5}+\frac{5}{3}$$
$$=\frac{(-24+25)}{15}=\frac{1}{15}$$

Example 7. *What should be subtracted from* $\frac{-5}{7}$ *to get* -1?

Solution: Let the required number be x. Then,

$$\frac{-5}{7}-x=-1 \quad\Rightarrow\quad \frac{-5}{7}=x-1$$
$$\Rightarrow\quad x=\left(\frac{-5}{7}+1\right)=\frac{(-5+7)}{7}=\frac{2}{7}$$

Hence, the required number is $\frac{2}{7}$.

Exercise 1C

1. Add the following rational numbers:

(a) $\frac{-7}{3}$ and $\frac{1}{3}$ (b) $\frac{5}{6}$ and $\frac{-1}{6}$ (c) $\frac{-2}{5}$ and $\frac{4}{5}$ (d) $\frac{-6}{11}$ and $\frac{-4}{11}$

(e) $\frac{-17}{15}$ and $\frac{-1}{15}$ (f) $\frac{-11}{8}$ and $\frac{5}{8}$

2. Add the following rational numbers:

(a) -1 and $\frac{3}{4}$ (b) 2 and $\frac{-5}{4}$ (c) 0 and $\frac{-2}{5}$ (d) $\frac{3}{4}$ and $\frac{-3}{5}$

(e) $\frac{5}{8}$ and $\frac{-7}{12}$ (f) $\frac{-8}{9}$ and $\frac{11}{6}$ (g) $\frac{1}{-12}$ and $\frac{2}{-15}$ (h) $\frac{-5}{16}$ and $\frac{7}{24}$

(i) $\frac{7}{-18}$ and $\frac{8}{27}$

3. Verify the following:

(a) $3+\frac{-7}{12}=\frac{-7}{12}+3$ (b) $\frac{-12}{5}+\frac{2}{7}=\frac{2}{7}+\frac{-12}{5}$

(c) $\frac{2}{-7}+\frac{12}{-35}=\frac{12}{-35}+\frac{2}{-7}$ (d) $\frac{-5}{8}+\frac{-9}{13}=\frac{-9}{13}+\frac{-5}{8}$

4. Verify the following:

(a) $-1+\left(\frac{-2}{3}+\frac{-3}{4}\right)=\left(-1+\frac{-2}{3}\right)+\frac{-3}{4}$ (b) $\left(\frac{3}{4}+\frac{-2}{5}\right)+\frac{-7}{10}=\frac{3}{4}+\left(\frac{-2}{5}+\frac{-7}{10}\right)$

(c) $\left(\frac{-7}{11}+\frac{2}{-5}\right)+\frac{-13}{22}=\frac{-7}{11}+\left(\frac{2}{-5}+\frac{-13}{22}\right)$

5. Fill in the blanks :

(a) $-9+\frac{-21}{8}=(\_\_\_)+(-9)$

(b) $\left(\frac{-3}{17}\right)+\left(\frac{-12}{5}\right)=\left(\frac{-12}{5}\right)+(\_\_\_)$

(c) $\frac{-16}{7}+(\_\_\_)=(\_\_\_)+\frac{-16}{7}=\frac{-16}{7}$

(d) $\frac{19}{-5}+\left(\frac{-3}{11}+\frac{-7}{8}\right)=\left\{\frac{19}{-5}+(\_\_\_)\right\}+\frac{-7}{8}$

(e) $-12+\left(\frac{7}{12}+\frac{-9}{11}\right)=\left(-12+\frac{7}{12}\right)+(\_\_\_)$

(f) $\left(\frac{-8}{13}+\frac{3}{7}\right)+\left(\frac{-13}{4}\right)=(\_\_\_)+\left[\frac{3}{7}+\left(\frac{-13}{4}\right)\right]$

6. Find the additive inverse of each of the following:

(a) $\frac{1}{5}$ (b) $\frac{21}{8}$ (c) -16 (d) $\frac{-15}{7}$

(e) $\frac{19}{-9}$ (f) $\frac{-24}{-15}$ (g) $\frac{-5}{13}$ (h) 0

(i) $\frac{29}{-16}$ (j) $\frac{-38}{-37}$

7. Subtract the following rational numbers:

(a) $\frac{-9}{7}$ from -1 (b) $\frac{-8}{9}$ from $\frac{-3}{5}$ (c) $\frac{-5}{6}$ from $\frac{1}{3}$ (d) $\frac{3}{4}$ from $\frac{1}{3}$

(e) -7 from $\frac{-4}{7}$ (f) $\frac{-13}{9}$ from 0 (g) $\frac{-18}{11}$ from 1 (h) $\frac{-32}{13}$ from $\frac{-6}{5}$

8. What number should be added to $\frac{-5}{8}$ so as to get $\frac{-3}{2}$?

9. The sum of two rational numbers is $\frac{-1}{2}$. If one of the numbers is $\frac{5}{6}$, find the other.

10. The sum of two rational numbers is -2. If one of the numbers is $\frac{-14}{5}$, find the other.

11. What number should be subtracted from $\frac{-2}{3}$ to get $\frac{-1}{6}$?

12. What number should be added to -1 so as to get $\frac{5}{7}$?

MULTIPLICATION OF RATIONAL NUMBERS

For any two rationals $\frac{p}{q}$ and $\frac{r}{s}$, we define

$$\left(\frac{p}{q}\times\frac{r}{s}\right)=\frac{(p\times r)}{(q\times s)}$$

Solved Examples

Example 1. *Find each of the following products:*

(*a*) $\frac{-7}{8}\times\frac{3}{5}$ (*b*) $\frac{-15}{4}\times\frac{-3}{8}$ (*c*) $\frac{2}{3}\times\frac{-5}{7}$

Solution: (a) $\frac{-7}{8}\times\frac{3}{5}=\frac{(-7)\times3}{8\times5}=\frac{-21}{40}$.

(b) $\frac{-15}{4}\times\frac{-3}{8}=\frac{(-15)\times(-3)}{4\times8}=\frac{45}{32}$.

(c) $\frac{2}{3}\times\frac{-5}{7}=\frac{2\times(-5)}{3\times7}=\frac{-10}{21}$

Example 2. *Find each of the following products:*

(*a*) $\frac{-3}{7}\times\frac{21}{5}$ (*b*) $\frac{-11}{9}\times\frac{-51}{44}$ (*c*) $\frac{13}{6}\times\frac{-18}{91}$

Solution: (a) $\frac{-3}{7}\times\frac{21}{5}=\frac{(-3)\times21}{7\times5}=\frac{-9}{5}$.

(b) $\frac{-11}{9}\times\frac{-51}{44}=\frac{(-11)\times(-51)}{9\times44}=\frac{11\times51}{9\times44}=\frac{17}{12}$.

(c) $\frac{13}{6}\times\frac{-18}{91}=\frac{13\times(-18)}{6\times91}=\frac{-(13\times18)}{(6\times91)}=\frac{-3}{7}$.

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

Property 1 (Closure Property): *The product of two rational numbers is always a rational number.*

If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers then $\left(\frac{p}{q}\times\frac{r}{s}\right)$ is also a rational number.

Examples. (a) Consider the rational numbers $\frac{1}{2}$ and $\frac{3}{5}$. Then,

$\left(\frac{1}{2}\times\frac{3}{5}\right)=\frac{(1\times3)}{(2\times5)}=\frac{3}{10}$, which is a rational number.

(b) Consider the rational numbers $\frac{-3}{7}$ and $\frac{5}{8}$. Then,

$\left(\frac{-3}{7}\times\frac{5}{8}\right)=\frac{(-3)\times5}{7\times8}=\frac{-15}{56}$, which is a rational number.

(c) Consider the rational numbers $\frac{-4}{5}$ and $\frac{-6}{5}$. Then,

$\left(\frac{-4}{5}\times\frac{-6}{5}\right)=\frac{(-4)\times(-6)}{5\times5}=\frac{24}{25}$, which is a rational number.

Property 2 (Commutative Law): *Two rational numbers can be multiplied in any order.*

Thus, for any rational numbers $\frac{p}{q}$ and $\frac{r}{s}$, we have

$$\left(\frac{p}{q}\times\frac{r}{s}\right)=\left(\frac{r}{s}\times\frac{p}{q}\right)$$

Examples. (a) Let us consider the rational numbers $\frac{-2}{5}$ and $\frac{6}{7}$. Then,

$$\left(\frac{-2}{5} \times \frac{6}{7}\right) = \frac{(-2) \times 6}{5 \times 7} = \frac{-12}{35} \quad \text{and} \quad \left(\frac{6}{7} \times \frac{-2}{5}\right) = \frac{6 \times (-2)}{7 \times 5} = \frac{-12}{35}$$

$$\therefore \quad \left(\frac{-2}{5} \times \frac{6}{7}\right) = \left(\frac{6}{7} \times \frac{-2}{5}\right)$$

(b) Let us consider the rational numbers $\frac{-2}{3}$ and $\frac{-5}{7}$. Then,

$$\left(\frac{-2}{3}\right) \times \left(\frac{-5}{7}\right) = \frac{(-2) \times (-5)}{3 \times 7} = \frac{10}{21} \quad \text{and} \quad \left(\frac{-5}{7}\right) \times \left(\frac{-2}{3}\right) = \frac{(-5) \times (-2)}{7 \times 3} = \frac{10}{21}$$

$$\therefore \quad \left(\frac{-2}{3}\right) \times \left(\frac{-5}{7}\right) = \left(\frac{-5}{7}\right) \times \left(\frac{-2}{3}\right)$$

(c) Let us consider the rational numbers $\frac{3}{4}$ and $\frac{5}{7}$. Then,

$$\left(\frac{3}{4} \times \frac{5}{7}\right) = \frac{(3 \times 5)}{(4 \times 7)} = \frac{15}{28} \quad \text{and} \quad \left(\frac{5}{7} \times \frac{3}{4}\right) = \frac{(5 \times 3)}{(7 \times 4)} = \frac{15}{28}$$

$$\therefore \quad \left(\frac{3}{4} \times \frac{5}{7}\right) = \left(\frac{5}{7} \times \frac{3}{4}\right)$$

Property 3 (Associative Law): *While multiplying three or more rational numbers, they can be grouped in any order.*

Thus, for any rationals $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{m}{n}$, we have

$$\left(\frac{p}{q} \times \frac{r}{s}\right) \times \frac{m}{n} = \frac{p}{q} \times \left(\frac{r}{s} \times \frac{m}{n}\right)$$

Example. Consider the rationals $\frac{-5}{2}$, $\frac{-7}{4}$ and $\frac{1}{3}$. We have

$$\left(\frac{-5}{2} \times \frac{-7}{4}\right) \times \frac{1}{3} = \left\{\frac{(-5) \times (-7)}{2 \times 4}\right\} \times \frac{1}{3} = \left(\frac{35}{8} \times \frac{1}{3}\right) = \frac{(35 \times 1)}{(8 \times 3)} = \frac{35}{24}$$

and
$$\frac{-5}{2} \times \left(\frac{-7}{4} \times \frac{1}{3}\right) = \frac{-5}{2} \times \frac{(-7) \times 1}{4 \times 3} = \left(\frac{-5}{2} \times \frac{-7}{12}\right) = \frac{(-5) \times (-7)}{(2 \times 12)} = \frac{35}{24}$$

$$\therefore \quad \left(\frac{-5}{2} \times \frac{-7}{4}\right) \times \frac{1}{3} = \frac{-5}{2} \times \left(\frac{-7}{4} \times \frac{1}{3}\right)$$

Property 4 (Existence of Multiplicative Identity): *For any rational number $\frac{p}{q}$, we have*

$$\left(\frac{p}{q} \times 1\right) = \left(1 \times \frac{p}{q}\right) = \frac{p}{q}$$

1 is called the ***multiplicative identity*** for rationals.

Examples. (a) Consider the rational numbers $\frac{-9}{13}$. Then, we have

$$\left(\frac{-9}{13} \times 1\right) = \left(\frac{-9}{13} \times \frac{1}{1}\right) = \frac{(-9) \times 1}{13 \times 1} = \frac{-9}{13} \text{ and } \left(1 \times \frac{-9}{13}\right) = \left(\frac{1}{1} \times \frac{-9}{13}\right) = \frac{1 \times (-9)}{1 \times 13} = \frac{-9}{13}$$

$$\therefore \quad \left(\frac{-9}{13} \times 1\right) = \left(1 \times \frac{-9}{13}\right) = \frac{-9}{13}$$

(b) Consider the rational numbers $\frac{3}{4}$. Then, we have

$$\left(\frac{3}{4}\times 1\right)=\left(\frac{3}{4}\times\frac{1}{1}\right)=\frac{(3\times 1)}{(4\times 1)}=\frac{3}{4} \text{ and } \left(1\times\frac{3}{4}\right)=\left(\frac{1}{1}\times\frac{3}{4}\right)=\frac{(1\times 3)}{(1\times 4)}=\frac{3}{4}$$

$$\therefore \quad \left(\frac{3}{4}\times 1\right)=\left(1\times\frac{3}{4}\right)=\frac{3}{4}$$

Property 5 (Existence of Multiplicative Inverse): *Every nonzero rational numbers $\frac{p}{q}$ has its multiplicative inverse $\frac{q}{p}$.*

Thus, $$\left(\frac{p}{q}\times\frac{q}{p}\right)=\left(\frac{q}{p}\times\frac{p}{q}\right)=1$$

$\frac{q}{p}$ *is called the* ***reciprocal*** *of* $\frac{p}{q}$.

Clearly, *zero has no reciprocal.*

Reciprocal of 1 *is* 1 *and the reciprocal of* (–1) *is* (–1).

Examples. (a) Reciprocal of $\frac{-8}{9}$ is $\frac{-9}{8}$, since $\left(\frac{-8}{9}\times\frac{-9}{8}\right)=\left(\frac{-9}{8}\times\frac{-8}{9}\right)=1.$

(b) Reciprocal of –3 is $\frac{-1}{3}$, since

$$\left(-3\times\frac{-1}{3}\right)=\left(\frac{-3}{1}\times\frac{-1}{3}\right)=\frac{(-3)\times(-1)}{1\times 3}=\frac{3}{3}=1$$

and $$\left(\frac{-1}{3}\times -3\right)=\left(\frac{-1}{3}\times\frac{-3}{1}\right)=\frac{(-1)\times(-3)}{3\times 1}=1.$$

(c) Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1.$

Remark

- We denote the reciprocal of $\frac{p}{q}$ by $\left(\frac{p}{q}\right)^{-1}$.

  Clearly, $$\left(\frac{p}{q}\right)^{-1}=\frac{q}{p}$$

Property 6 (Distributive Law of Multiplication over Addition): *For any three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{m}{n}$, we have*

$$\frac{p}{q}\times\left(\frac{r}{s}+\frac{m}{n}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)+\left(\frac{p}{q}\times\frac{m}{n}\right)$$

Example. Consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$. We have

$$\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\right)\times\left\{\frac{4+(-5)}{6}\right\}=\left(\frac{-3}{4}\right)\times\left(\frac{-1}{6}\right)=\frac{(-3)\times(-1)}{4\times 6}=\frac{3}{24}=\frac{1}{8}.$$

Again, $\left(\frac{-3}{4}\right)\times\frac{2}{3}=\frac{(-3)\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$ and $\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)=\frac{(-3)\times(-5)}{4\times 6}=\frac{15}{24}=\frac{5}{8}$.

$\therefore$ $\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}=\left(\frac{-1}{2}+\frac{5}{8}\right)=\frac{(-4+5)}{8}=\frac{1}{8}$.

Hence, $\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}$.

Property 7 (Multiplicative Property of 0): *Every rational number multiplied with* 0 *gives* 0.

Thus, for any rational number $\frac{p}{q}$, we have

$$\left(\frac{p}{q}\times 0\right)=\left(0\times\frac{p}{q}\right)=0.$$

Examples. (a) $\left(\frac{-12}{17}\times 0\right)=\left(\frac{-12}{17}\times\frac{0}{1}\right)=\frac{(-12)\times 0}{17\times 1}=\frac{0}{17}=0$. Similarly, $\left(0\times\frac{-12}{17}\right)=0$.

(b) $\left(\frac{5}{18}\times 0\right)=\left(\frac{5}{18}\times\frac{0}{1}\right)=\frac{(5\times 0)}{(18\times 1)}=\frac{0}{18}=0$. Similarly, $\left(0\times\frac{5}{18}\right)=0$.

Solved Examples

Example 1. *Find the reciprocal of each of the following:*

(*a*) 15 (*b*) −6 (*c*) $\frac{4}{13}$ (*d*) $\frac{-17}{19}$

Solution: (a) Reciprocal of 15 is $\frac{1}{15}$.

(b) Reciprocal of −6 is $\frac{1}{-6}$, *i.e.*, $\frac{-1}{6}$.

(c) Reciprocal of $\frac{4}{13}$ is $\frac{13}{4}$.

(d) Reciprocal of $\frac{-17}{19}$ is $\frac{19}{-17}$, *i.e.*, $\frac{-19}{17}$.

Example 2. *Verify that:*

(*a*) $\left(\frac{-3}{16}\times\frac{8}{15}\right)=\left(\frac{8}{15}\times\frac{-3}{16}\right)$

(*b*) $\frac{2}{3}\times\left(\frac{6}{7}+\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$

(*c*) $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$

Solution: (a) LHS $=\left(\frac{-3}{16}\times\frac{8}{15}\right)=\frac{(-3)\times 8}{16\times 15}=\frac{-24}{240}=\frac{-1}{10}$.

RHS $=\left(\frac{8}{15}\times\frac{-3}{16}\right)=\frac{8\times(-3)}{15\times 16}=\frac{-24}{240}=\frac{-1}{10}$.

$\therefore$ LHS = RHS.

Hence, $\left(\frac{-3}{16}\times\frac{8}{15}\right)=\left(\frac{8}{15}\times\frac{-3}{16}\right)$.

(b) LHS $= \frac{2}{3}\times\left(\frac{6}{7}+\frac{-14}{15}\right)=\frac{2}{3}\times\frac{6\times(-14)}{7\times15}=\frac{2}{3}\times\frac{-84}{105}$

$=\frac{2}{3}\times\frac{-4}{5}=\frac{2\times(-4)}{3\times5}=\frac{-8}{15}.$

RHS $=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}=\frac{(2\times6)}{(3\times7)}\times\frac{-14}{15}=\frac{12}{21}\times\frac{-14}{15}$

$=\frac{4}{7}\times\frac{-14}{15}=\frac{4\times(-14)}{(7\times15)}=\frac{-56}{105}=\frac{-8}{15}.$

$\therefore$ LHS = RHS.

Hence, $\frac{2}{3}\times\left(\frac{6}{7}+\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}.$

(c) LHS $=\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\frac{5}{6}\times\left[\frac{(-8)+(-7)}{10}\right]=\frac{5}{6}\times\frac{-15}{10}$

$=\frac{5}{6}\times\frac{-3}{2}=\frac{5\times(-3)}{6\times2}=\frac{-15}{12}=\frac{-5}{4}.$

RHS $=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)=\frac{5\times(-4)}{6\times5}+\frac{5\times(-7)}{6\times10}=\frac{-20}{30}+\frac{-35}{60}$

$=\frac{-2}{3}+\frac{-7}{12}=\frac{(-8)+(-7)}{12}=\frac{-15}{12}=\frac{-5}{4}.$

$\therefore$ LHS = RHS.

Hence, $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right).$

Exercise 1D

1. Find each of the following products:

(a) $\frac{-2}{3}\times\frac{6}{7}$ (b) $\frac{-12}{5}\times\frac{10}{-3}$ (c) $\frac{25}{-9}\times\frac{3}{-10}$ (d) $\frac{3}{5}\times\frac{-7}{8}$

(e) $\frac{-9}{2}\times\frac{5}{4}$ (f) $\frac{-6}{11}\times\frac{-5}{3}$ (g) $\frac{16}{-21}\times\frac{14}{5}$ (h) $\frac{5}{-18}\times\frac{-9}{20}$

(i) $\frac{-13}{15}\times\frac{-25}{26}$ (j) $\frac{7}{24}\times(-48)$ (k) $\frac{-13}{5}\times(-10)$ (*l*) $\frac{-7}{6}\times24$

2. Find the multiplicative inverse (*i.e.,* reciprocal) of:

(a) $\frac{17}{27}$ (b) $\frac{-15}{11}$ (c) $\frac{-6}{25}$ (d) 21

(e) −19 (f) $\frac{-2}{-7}$ (g) $\frac{0}{5}$ (h) −1

(i) $\frac{3}{-11}$ (j) $\frac{-2}{9}$

3. Verify each of the following:

(a) $-8\times\frac{-13}{12}=\frac{-13}{12}\times(-8)$ (b) $\frac{-12}{5}\times\frac{7}{-36}=\frac{7}{-36}\times\frac{-12}{5}$

(c) $\frac{-8}{7} \times \frac{13}{9} = \frac{13}{9} \times \frac{-8}{7}$

(d) $\frac{3}{7} \times \frac{-5}{9} = \frac{-5}{9} \times \frac{3}{7}$

4. Verify each of the following:

(a) $\frac{-13}{24} \times \left(\frac{-12}{5} \times \frac{35}{36}\right) = \left(\frac{-13}{24} \times \frac{-12}{5}\right) \times \frac{35}{36}$

(b) $\left(\frac{-9}{5} \times \frac{-10}{3}\right) \times \frac{21}{-4} = \frac{-9}{5} \times \left(\frac{-10}{3} \times \frac{21}{-4}\right)$

(c) $\left(\frac{5}{7} \times \frac{12}{13}\right) \times \frac{7}{18} = \frac{5}{7} \times \left(\frac{12}{13} \times \frac{7}{18}\right)$

5. Fill in the blanks:

(a) $\frac{-23}{17} \times \frac{18}{35} = \frac{18}{35} \times (\_\_\_)$

(b) $-38 \times \frac{-7}{19} = \frac{-7}{19} \times (\_\_\_)$

(c) $\left(\frac{15}{7} \times \frac{-21}{10}\right) \times \frac{-5}{6} = (\_\_\_) \times \left(\frac{-21}{10} \times \frac{-5}{6}\right)$

(d) $\frac{-12}{5} \times \left(\frac{4}{15} \times \frac{25}{-16}\right) = \left(\frac{-12}{5} \times \frac{4}{15}\right) \times (\_\_\_)$

6. Find the value of:

(a) $\left(\frac{3}{7}\right)^{-1}$ (b) $\left(\frac{-5}{8}\right)^{-1}$ (c) $(-11)^{-1}$ (d) $\left(\frac{1}{-7}\right)^{-1}$

7. Verify the following:

(a) $\left(\frac{-8}{3} + \frac{-13}{12}\right) \times \frac{5}{6} = \left(\frac{-8}{3} \times \frac{5}{6}\right) + \left(\frac{-13}{12} \times \frac{5}{6}\right)$

(b) $\frac{3}{7} \times \left(\frac{5}{6} + \frac{12}{13}\right) = \left(\frac{3}{7} \times \frac{5}{6}\right) + \left(\frac{3}{7} \times \frac{12}{13}\right)$

(c) $\frac{-16}{7} \times \left(\frac{-8}{9} + \frac{-7}{6}\right) = \left(\frac{-16}{7} \times \frac{-8}{9}\right) + \left(\frac{-16}{7} \times \frac{-7}{6}\right)$

(d) $\frac{-15}{4} \times \left(\frac{3}{7} + \frac{-12}{5}\right) = \left(\frac{-15}{4} \times \frac{3}{7}\right) + \left(\frac{-15}{4} \times \frac{-12}{5}\right)$

8. Name the property of multiplication illustrated by each of the following statements:

(a) $\frac{-8}{3} \times 0 = 0$

(b) $\frac{-13}{17} \times \frac{17}{-13} = \frac{17}{-13} \times \frac{-13}{17} = 1$

(c) $\frac{-19}{7} \times 1 = 1 \times \frac{-19}{7} = \frac{-19}{7}$

(d) $\frac{-2}{5} \times \left(\frac{-6}{7} + \frac{8}{9}\right) = \left(\frac{-2}{5} \times \frac{-6}{7}\right) + \left(\frac{-2}{5} \times \frac{8}{9}\right)$

(e) $\left(\frac{-3}{4} \times \frac{5}{6}\right) \times \frac{-7}{8} = \frac{-3}{4} \times \left(\frac{5}{6} \times \frac{-7}{8}\right)$

(f) $\frac{-11}{5} \times \frac{-13}{9} = \frac{-13}{9} \times \frac{-11}{5}$

DIVISION OF RATIONAL NUMBERS

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that $\frac{r}{s} \neq 0$, we define,

$$\left(\frac{p}{q} \div \frac{r}{s}\right) = \left(\frac{p}{q} \times \frac{s}{r}\right)$$

When $\frac{p}{q}$ is divided by $\frac{r}{s}$ then $\frac{p}{q}$ called the ***dividend***; $\frac{r}{s}$ is called the ***divisor*** and the result is known as ***quotient***.

Solved Examples

Example 1. *Divide:*

(*a*) $\frac{-9}{40}$ *by* $\frac{-3}{8}$ (*b*) $\frac{11}{24}$ *by* $\frac{-5}{8}$ (*c*) $\frac{-6}{25}$ *by* $\frac{3}{5}$ (*d*) $\frac{9}{16}$ *by* $\frac{5}{8}$

Solution: (a) $\frac{-9}{40} \div \frac{-3}{8} = \frac{-9}{40} \times \frac{8}{-3} = \frac{(-9)\times 8}{40\times(-3)} = \frac{-72}{-120} = \frac{3}{5}$.

(b) $\frac{11}{24} \div \frac{-5}{8} = \frac{11}{24} \times \frac{8}{-5} = \frac{11\times 8}{24\times(-5)} = \frac{88}{-120} = \frac{-11}{15}$.

(c) $\frac{-6}{25} \div \frac{3}{5} = \frac{-6}{25} \times \frac{5}{3} = \frac{(-6)\times 5}{25\times 3} = \frac{-30}{75} = \frac{-2}{5}$.

(d) $\frac{9}{16} \div \frac{5}{8} = \frac{9}{16} \times \frac{8}{5} = \frac{9\times 8}{16\times 5} = \frac{72}{80} = \frac{9}{10}$.

Example 2. *Fill in the blanks:* $\frac{27}{16} \div (\_\_\_) = \frac{-15}{8}$.

Solution: Let $\frac{27}{16} \div \left(\frac{p}{q}\right) = \frac{-15}{8}$. Then,

$\frac{27}{16} \times \frac{q}{p} = \frac{-15}{8}$ $\Rightarrow$ $\frac{q}{p} = \frac{-15}{8} \times \frac{16}{27} = \frac{-10}{9}$

$\Rightarrow$ $\frac{p}{q} = \frac{9}{-10} = \frac{-9}{10}$.

Hence, the missing number is $\frac{-9}{10}$.

PROPERTIES OF DIVISION

Property 1 (Closure Property): *If* $\frac{p}{q}$ *and* $\frac{r}{s}$ *are any two rational numbers such that* $\frac{r}{s} \neq 0$ *then* $\left(\frac{p}{q} \div \frac{r}{s}\right)$ *is also a rational number.*

Property 2 (Property of 1): *For every rational number* $\frac{p}{q}$, *we have:*

$$\left(\frac{p}{q} \div 1\right) = \frac{p}{q}$$

Property 3: *For every nonzero rational number* $\frac{p}{q}$, *we have:*

$$\left(\frac{p}{q} \div \frac{p}{q}\right) = 1$$

Exercise 1E

1. Simplify:

(a) $-8 \div \frac{-7}{16}$ (b) $\frac{-12}{7} \div (-18)$ (c) $\frac{-1}{10} \div \frac{-8}{5}$ (d) $\frac{4}{9} \div \frac{-5}{12}$

(e) $\frac{-65}{14} \div \frac{13}{7}$ (f) $\frac{-16}{35} \div \frac{-15}{14}$

2. Verify whether the given statement is true or false:

(a) $\left\{(-16)\div\frac{6}{5}\right\}\div\frac{-9}{10}=(-16)\div\left\{\frac{6}{5}\div\frac{-9}{10}\right\}$ (b) $\left(\frac{-3}{5}\div\frac{-12}{35}\right)\div\frac{1}{14}=\frac{-3}{5}\div\left(\frac{-12}{35}\div\frac{1}{14}\right)$

(c) $\left(\frac{5}{9}\div\frac{1}{3}\right)\div\frac{5}{2}=\frac{5}{9}\div\left(\frac{1}{3}\div\frac{5}{2}\right)$

3. Verify whether the given statement is true or false:

(a) $-9\div\frac{3}{4}=\frac{3}{4}\div(-9)$ (b) $\frac{-8}{9}\div\frac{-4}{3}=\frac{-4}{3}\div\frac{-8}{9}$

(c) $\frac{-7}{24}\div\frac{3}{-16}=\frac{3}{-16}\div\frac{-7}{24}$ (d) $\frac{13}{5}\div\frac{26}{10}=\frac{26}{10}\div\frac{13}{5}$

4. The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.

5. The product of two rational numbers is –9. If one of the numbers is –12, find the other.

6. By what rational number should $\frac{-8}{39}$ be multiplied to obtain $\frac{1}{26}$?

7. By what rational number should we multiply $\frac{-15}{56}$ to get $\frac{-5}{7}$?

8. By what number should $\frac{-33}{8}$ be divided to get $\frac{-11}{2}$?

9. Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.

10. Fill in the blanks:

(a) $(-12)\div(\_\_\_)=\frac{-6}{5}$ (b) $(\_\_\_)\div(-3)=\frac{-4}{15}$

(c) $(\_\_\_)\div\left(\frac{-7}{5}\right)=\frac{10}{19}$ (d) $\frac{9}{8}\div(\_\_\_)=\frac{-3}{2}$

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

We already know that there is an infinite number of rational numbers between any two given rational numbers.

As a general rule, if a and b are two rational numbers such that $a < b$, than $\frac{a+b}{2}$ is a rational number lying between a and b.

There are two methods of finding rational numbers.

METHOD I.

1. Add the two rational numbers by taking LCM.
2. Divide the sum by 2. The result is a rational number, which lies in between the two rational numbers.

Let $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers, such that $\frac{p}{q}<\frac{r}{s}$, then $\frac{1}{2}\left(\frac{p}{q}+\frac{r}{s}\right)$ is a rational number lying between $\frac{p}{q}$ and $\frac{r}{s}$.

Example 1. *Find a rational number between* $\frac{2}{5}$ *and* $\frac{7}{10}$.

Solution: Rational number between $\frac{2}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{2}{5}+\frac{7}{10}\right) = \frac{1}{2}\left(\frac{2\times 2+7\times 1}{10}\right)$

$$= \frac{1}{2}\left(\frac{4+7}{10}\right) = \frac{1}{2}\times\frac{11}{10} = \frac{11}{20}$$

METHOD II. By making like terms

1. Convert the denominators of both the fractions into the same denominator by taking their LCM.
2. If the new fractions do not have any number in between the given numerators, then multiply them as well as denominators of both fractions by 10 or is multiple.

Example 2. *Find any five rational numbers between* $\frac{1}{12}$ *and* $\frac{5}{18}$.

Solution: Convert $\frac{1}{12}$ and $\frac{5}{18}$ into like terms.

So, $\frac{1}{12} = \frac{1\times 3}{12\times 3} = \frac{3}{36}$

and $\frac{5}{18} = \frac{5\times 2}{18\times 2} = \frac{10}{36}$ (LCM of 12 and 18 is 36)

2	12, 18
2	6, 9
3	3, 9
3	1, 3
	1, 1

So, LCM $= 2\times 2\times 3\times 3$
$= 36$

So, five rational numbers between $\frac{3}{36}$ and $\frac{10}{36}$

are $\frac{4}{36}, \frac{5}{36}, \frac{6}{36}, \frac{7}{36}$ and $\frac{8}{36}$.

Example 3. *Insert* 10 *rational numbers between* –3 *and* 2.

Solution: –3 is written as $\frac{-3\times 10}{1\times 10} = \frac{-30}{10}$

and 2 is written as $\frac{2\times 10}{1\times 10} = \frac{20}{10}$

So, 10 rational numbers between $\frac{-30}{10}$ and $\frac{20}{10}$ are $\frac{-29}{10}, \frac{-28}{10}, \frac{-27}{10}, \frac{-26}{10}, \frac{-25}{10}, \frac{-24}{10}, \frac{-23}{10}, \frac{-22}{10}, \frac{-21}{10}$ and $\frac{-20}{10}$.

Example 4. *Find* 20 *rational numbers between* $\frac{-5}{6}$ *and* $\frac{5}{8}$.

Solution: LCM of 6 and 8 is 24.

Now, $\frac{-5}{6} = \frac{-5\times 4}{6\times 4} = \frac{-20}{24}$ and $\frac{5}{8} = \frac{5\times 3}{8\times 3} = \frac{15}{24}$.

Rational numbers lying between $\frac{-5}{6}$ and $\frac{5}{8}$ are

$$\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \frac{-16}{24}, \ldots, \frac{-1}{24}, \frac{0}{24}, \frac{1}{24}, \frac{2}{24}, \frac{3}{24}, \ldots, \frac{14}{24}.$$

Out of these 20 may be taken.

Example 5. *Find 15 rational numbers between –2 and 0.*

Solution: We may write, $-2 = \frac{-20}{10}$ and $0 = \frac{0}{10}$.

Rational numbers lying between –2 and 0 are $\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \frac{-14}{10}, \frac{-13}{10},$
$\frac{-12}{10}, \frac{-11}{10}, -1, \frac{-9}{10}, \frac{-8}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}, \frac{-4}{10}, \frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}.$

Out of these 15 may be taken.

Example 6. *Write 9 rational numbers between 1 and 2.*

Solution: We may write $1 = \frac{10}{10}$ and $2 = \frac{20}{10}$

$\therefore$ rational numbers between 1 and 2 are

$$\frac{11}{10}, \frac{12}{10}, \frac{13}{10}, \frac{14}{10}, \frac{15}{10}, \frac{16}{10}, \frac{17}{10}, \frac{18}{10}, \frac{19}{10}.$$

Remark

➲ *Suppose we have to write 99 rational numbers between 1 and 2.*

Then, we may write, $1 = \frac{100}{100}$ *and* $2 = \frac{200}{100}$.

$\therefore$ *rational numbers between 1 and 2 are*

$$\frac{101}{100}, \frac{102}{100}, \frac{103}{100}, \ldots, \frac{198}{100}, \frac{199}{100}.$$

Exercise 1F

1. Find a rational number between $\frac{-1}{3}$ and $\frac{1}{2}$.
2. Find a rational number between $\frac{1}{4}$ and $\frac{1}{3}$.
3. Find a rational number between 2 and 3.
4. Find three rational numbers between 4 and 5.
5. Find three rational numbers between $\frac{2}{3}$ and $\frac{3}{4}$.
6. Find two rational numbers between –3 and –2.
7. Find 12 rational numbers between –1 and 2.
8. Find 10 rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$.

WORD PROBLEMS

Example 1. *Seema bought* $5\frac{1}{4}$ kg *of potatoes,* $3\frac{1}{5}$ kg *of tomatoes and* $2\frac{4}{5}$ kg *of onions. Find the total weight of the vegetables.*

Solution: Weight of potatoes $= 5\frac{1}{4}$ kg

Weight of tomatoes $= 3\frac{1}{5}$ kg

Weight of onions $= 2\frac{4}{5}$ kg

So, total weight of the vegetables $= \left(5\frac{1}{4} + 3\frac{1}{5} + 2\frac{4}{5}\right)\text{kg} = \left(\frac{21}{4} + \frac{16}{5} + \frac{14}{5}\right)\text{kg}$

$$= \left(\frac{21\times 5 + 16\times 4 + 14\times 4}{20}\right)\text{kg} = \frac{105 + 64 + 56}{20}\text{ kg}$$

$$= \frac{225}{20}\text{ kg} = \frac{45}{4}\text{ kg} = 11\frac{1}{4}\text{ kg}$$

Hence, the total weight of vegetables is $11\frac{1}{4}$ kg.

Example 2. *A drum contains* $90\frac{1}{4}l$ *of oil. If* $15\frac{1}{5}l$ *of oil is taken out from it, find the quantity of remaining oil.*

Solution: Total quantity of oil in drum $= 90\frac{1}{4}l$

Quantity of oil taken from it $= 15\frac{1}{5}l$

So, remaining quantity of oil $= \left(90\frac{1}{4} - 15\frac{1}{5}\right)l$

$$= \left(\frac{361}{4} - \frac{76}{5}\right)l = \left(\frac{361\times 5 - 76\times 4}{20}\right)l$$

$$= \left(\frac{1805 - 304}{20}\right)l = \frac{1501}{20}l = 75\frac{1}{20}l$$

Hence, $75\frac{1}{20}l$ of oil remains in the drum.

Example 3. *A train covers a distance of* $70\frac{4}{5}$ km *in* 1 *hour. How much distance will it cover in* $3\frac{3}{4}$ *hours*?

Solution: Distance covered in 1 hour $= 70\frac{4}{5}$ km

$\therefore$ Distance covered in $3\frac{3}{4}$ hours $= 70\frac{4}{5} \times 3\frac{3}{4}$ km

$$= \frac{354}{5} \times \frac{15}{4} = \frac{354\times 15}{5\times 4}\text{ km}$$

$$= \frac{177\times 3}{2} = \frac{531}{2} = 265\frac{1}{2}\text{ km}$$

Example 4. $82\frac{1}{4}$ kg *rice is to be divided among* 12 *persons equally. How much rice will each person get?*

Solution: Total quantity of rice $= 82\frac{1}{4}$ kg

Number of persons $= 12$

So, each person will get $= \left(82\frac{1}{4} \div 12\right)$ kg

$= \left(\frac{329}{4} \times \frac{1}{12}\right)$ kg

$= \frac{329}{48}$ kg $= 6\frac{41}{48}$ kg

Hence, each person will get $6\frac{41}{48}$ kg of rice.

Exercise 1G

1. A basket contains three types of fruits weighing $19\frac{1}{3}$ kg in all. If $8\frac{1}{9}$ kg of these be apples, $3\frac{1}{6}$ kg be oranges and the rest pears, what is the weight of the pears in the basket?

2. From a wire 11 m long, two pieces of lengths $2\frac{3}{5}$ m and $3\frac{3}{10}$ m are cut off. What is the length of the remaining wire?

3. On one day a rickshaw puller earned ₹ 160. Out of his earnings he spent ₹ $26\frac{3}{5}$ on tea and snacks, ₹ $50\frac{1}{2}$ on food and ₹ $16\frac{2}{5}$ on repairs of the rickshaw. How much did he save on that day?

4. A drum full of sugar weighs $40\frac{1}{6}$ kg. If the empty drum weighs $13\frac{3}{4}$ kg, find the weight of sugar in the drum.

5. Find the area of a rectangular park which is $36\frac{3}{5}$ m long and $16\frac{2}{3}$ m broad.

6. Find the area of a square plot of land whose each side measures $8\frac{1}{2}$ metres.

7. One litre of petrol costs ₹ $63\frac{3}{4}$. What is the cost of 34 litres of petrol?

8. An aeroplane covers 1020 km in an hour. How much distance will it cover in $4\frac{1}{6}$ hours?

9. Find the cost of $3\frac{2}{5}$ metres of cloth at ₹ $63\frac{3}{4}$ per metre.

10. A car is moving at an average speed of $60\frac{2}{5}$ km/hr. How much distance will it cover in $6\frac{1}{4}$ hours?

11. In a school, $\frac{5}{8}$ of the students are boys. If there are 240 girls, find the number of boys in the school.

12. The product of two fractions is $9\frac{3}{5}$. If one of the fractions is $9\frac{3}{7}$, find the other.

13. The cost of $3\frac{1}{2}$ metres of cloth is ₹ $166\frac{1}{4}$. What is the cost of one metre of cloth?

14. A rope of length $71\frac{1}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?

15. The area of a room is $65\frac{1}{4}$ m^2. If its breadth is $5\frac{7}{16}$ metres, what is its length?

Exercise 1H

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. $\left(\frac{31}{-4}+\frac{-5}{8}\right)=?$

   (a) $\frac{67}{8}$ (b) $\frac{57}{8}$ (c) $\frac{-57}{8}$ (d) $\frac{-67}{8}$

2. $\left(3+\frac{5}{-7}\right)=?$

   (a) $\frac{-16}{7}$ (b) $\frac{16}{7}$ (c) $\frac{-26}{7}$ (d) $\frac{-8}{7}$

3. $\left(\frac{-5}{16}+\frac{7}{12}\right)=?$

   (a) $-\frac{7}{48}$ (b) $\frac{1}{24}$ (c) $\frac{13}{48}$ (d) $\frac{1}{3}$

4. $\left(\frac{8}{-15}+\frac{4}{-3}\right)=?$

   (a) $\frac{28}{15}$ (b) $\frac{-28}{15}$ (c) $\frac{-4}{5}$ (d) $\frac{-4}{15}$

5. $\left(\frac{7}{-26}+\frac{16}{39}\right)=?$

   (a) $\frac{11}{78}$ (b) $\frac{-11}{78}$ (c) $\frac{11}{39}$ (d) $\frac{-11}{39}$

6. What should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

   (a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) $\frac{-5}{2}$

7. What should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

   (a) $\frac{-29}{21}$ (b) $\frac{29}{21}$ (c) $\frac{1}{21}$ (d) $\frac{-1}{21}$

8. What should be added to $\frac{7}{12}$ to get $\frac{-4}{15}$?

   (a) $\frac{17}{20}$ (b) $\frac{-17}{20}$ (c) $\frac{7}{20}$ (d) $\frac{-7}{20}$

9. The sum of two numbers is $\frac{-4}{3}$. If one of the numbers is –5, what is the other?

(a) $\frac{-11}{3}$ (b) $\frac{11}{3}$ (c) $\frac{-19}{3}$ (d) $\frac{19}{3}$

10. $\left(\frac{-3}{7}\right)^{-1} = ?$

(a) $\frac{7}{3}$ (b) $\frac{-7}{3}$ (c) $\frac{3}{7}$ (d) none of these

11. The product of two numbers is $\frac{-16}{35}$. If one of the numbers is $\frac{-15}{14}$, the other is:

(a) $\frac{-2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{32}{75}$ (d) $\frac{-8}{3}$

12. The product of two numbers is $\frac{-28}{81}$. If one of the numbers is $\frac{14}{27}$ then the other one is:

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

13. What should be subtracted from $\frac{-3}{5}$ to get –2 ?

(a) $\frac{-7}{5}$ (b) $\frac{-13}{5}$ (c) $\frac{13}{5}$ (d) $\frac{7}{5}$

14. $\left(\frac{-9}{16} \times \frac{8}{15}\right) = ?$

(a) $\frac{-3}{10}$ (b) $\frac{-4}{15}$ (c) $\frac{-9}{25}$ (d) $\frac{-2}{5}$

15. $\left(\frac{-5}{9} \div \frac{2}{3}\right) = ?$

(a) $\frac{-5}{2}$ (b) $\frac{-5}{6}$ (c) $\frac{-10}{27}$ (d) $\frac{-6}{5}$

16. $\frac{4}{9} \div ? = \frac{-8}{15}$

(a) $\frac{-32}{45}$ (b) $\frac{-8}{5}$ (c) $\frac{-9}{10}$ (d) $\frac{-5}{6}$

17. Which of the following numbers is in standard form?

(a) $\frac{-12}{26}$ (b) $\frac{-49}{70}$ (c) $\frac{-9}{16}$ (d) $\frac{28}{-105}$

18. Additive inverse of $\frac{-5}{9}$ is:

(a) $\frac{-9}{5}$ (b) 0 (c) $\frac{5}{9}$ (d) $\frac{9}{5}$

19. Reciprocal of $\frac{-3}{4}$ is:

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) 0

20. A rational number which does not lie between $\frac{-2}{3}$ and $\frac{1}{4}$ is:

(a) $\frac{5}{12}$ (b) $\frac{-5}{12}$ (c) $\frac{5}{24}$ (d) $\frac{-5}{24}$

THINGS TO REMEMBER

- The numbers of the form $\frac{p}{q}$, where a and b are integers and $b \neq 0$, are called rational numbers.
- (i) A rational number is said to be positive if its numerator and denominator are either both positive or both negative.
  (ii) A rational number is said to be negative if its numerator and denominator are of opposite signs.
- (i) If $\frac{p}{q}$ is a rational number and m is a nonzero integer then $\frac{p}{q} = \frac{p \times m}{q \times m}$.
  (ii) If $\frac{p}{q}$ is a rational number and m is a common divisor of both a and b then $\frac{p}{q} = \frac{p \div m}{q \div m}$.
- A rational number $\frac{p}{q}$ is said to be in standard form if p and q are integers having no common divisor other than 1 and q is positive.
- $\frac{p}{q} = \frac{r}{s}$ only when $(p \times s) = (q \times r)$.
- To compare two or more rational numbers, express each of them as rational number with positive denominator. Take the LCM of these positive denominators and express each rational number with this LCM as denominator. Then, the number having the greater numerator is greater.
- If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers then

  (i) $\left(\frac{p}{q} + \frac{r}{s}\right)$ is also a rational number. [closure property]

  (ii) $\frac{p}{q} + \frac{r}{s} = \frac{r}{s} + \frac{p}{q}$ [commutative law of addition]

  (iii) $\left(\frac{p}{q} + \frac{r}{s}\right) + \frac{m}{n} = \frac{p}{q} + \left(\frac{r}{s} + \frac{m}{n}\right)$ [associative law of addition]

  (iv) $\frac{p}{q} + 0 = 0 + \frac{p}{q} = \frac{p}{q}$

  0 is called the identity element for addition of rational numbers.

(v) $\left(\frac{p}{q}+\frac{-p}{q}\right)=\left(\frac{-p}{q}+\frac{p}{q}\right)=0$

$\frac{-p}{q}$ is called the additive inverse of $\frac{p}{q}$.

- If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers then $\frac{p}{q}\times\frac{r}{s}=\frac{p\times r}{q\times s}$.
- If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers then

(i) $\left(\frac{p}{q}\times\frac{r}{s}\right)$ is also a rational number. [closure property]

(ii) $\left(\frac{p}{q}\times\frac{r}{s}\right)=\left(\frac{r}{s}\times\frac{p}{q}\right)$ [commutative law of multiplication]

(iii) $\left(\frac{p}{q}\times\frac{r}{s}\right)\times\frac{m}{n}=\frac{p}{q}\times\left(\frac{r}{s}\times\frac{m}{n}\right)$ [associative law of multiplication]

(iv) $\left(\frac{p}{q}\times 1\right)=\left(1\times\frac{p}{q}\right)=\frac{p}{q}$

1 is called the multiplicative identity for rationals.

(v) $\left(\frac{p}{q}\times\frac{q}{p}\right)=\left(\frac{q}{p}\times\frac{p}{q}\right)=1$

$\frac{q}{p}$ is called the multiplicative inverse or reciprocal of $\frac{p}{q}$.

(vi) $\frac{p}{q}\times\left(\frac{r}{s}+\frac{m}{n}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)+\left(\frac{p}{q}\times\frac{m}{n}\right)$ [distributive law]

(vii) $\left(\frac{p}{q}\times 0\right)=\left(0\times\frac{p}{q}\right)=0$

- If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that $\frac{r}{s}\neq 0$ then $\left(\frac{p}{q}\div\frac{r}{s}\right)=\left(\frac{p}{q}\times\frac{s}{r}\right)$.

(i) If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers and $\frac{r}{s}\neq 0$ then $\left(\frac{p}{q}\div\frac{r}{s}\right)$ is also a rational numbers.

(ii) For every rational number $\frac{p}{q}$, we have $\left(\frac{p}{q}\div 1\right)=\frac{p}{q}$ and $\left(\frac{p}{q}\div\frac{p}{q}\right)=1$.

- If a and b be two rational numbers such that $a<b$ then $\frac{1}{2}(a+b)$ is a rational number between a and b.

2 Exponents

We have already learnt about exponents in previous class. In this chapter, we shall learn some more rules of exponents and negative exponents.

Let us recall that for positive integers a and n, we have

$$(-a)^n = \begin{cases} a^n, \text{ when } n \text{ is even.} \\ -a^n, \text{ when } n \text{ is odd.} \end{cases}$$

Examples. (a) $(-3)^4 = (-3)\times(-3)\times(-3)\times(-3) = 81 = 3^4$

(b) $(-3)^3 = (-3)\times(-3)\times(-3) = -27 = -3^3$

Positive Integral Exponent of a Rational Number

Let $\frac{a}{b}$ be any rational number and n be a positive integer. Then,

$$\left(\frac{a}{b}\right)^n = \frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\ldots n \text{ times} = \frac{a\times a\times a\times\ldots n \text{ times}}{b\times b\times b\times\ldots n \text{ times}} = \frac{a^n}{b^n}$$

Thus, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ for every positive integer n.

Example 1. *Evaluate:*

(*a*) $\left(\frac{2}{3}\right)^3$ (*b*) $\left(\frac{-3}{5}\right)^4$ (*c*) $\left(\frac{-2}{3}\right)^5$

Solution: We have:

(a) $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

(b) $\left(\frac{-3}{5}\right)^4 = \frac{(-3)^4}{(5)^4} = \frac{3^4}{5^4} = \frac{81}{625}$

(c) $\left(\frac{-2}{3}\right)^5 = \frac{(-2)^5}{3^5} = \frac{-2^5}{3^5} = \frac{-32}{243}$

Negative Integral Exponent of a Rational Number

Let $\frac{a}{b}$ be any rational number and n be a positive integer.

Then, we define, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Examples. (a) $4^{-6} = \left(\frac{4}{1}\right)^{-6} = \left(\frac{1}{4}\right)^6$ (b) $\left(\frac{3}{4}\right)^{-5} = \left(\frac{4}{3}\right)^5$

If $\frac{a}{b}$ be any rational number, then $\left(\frac{a}{b}\right)^0 = 1$.

Example 2. *Evaluate:*

(*a*) 4^{-2} (*b*) $\left(\frac{1}{6}\right)^{-2}$ (*c*) $\left(\frac{2}{3}\right)^{-3}$ (*d*) $\left(\frac{2}{3}\right)^{0}$

Solution: (a) $4^{-2}=\left(\frac{4}{1}\right)^{-2}=\left(\frac{1}{4}\right)^{2}=\frac{1^2}{4^2}=\frac{1}{16}$.

(b) $\left(\frac{1}{6}\right)^{-2}=\left(\frac{6}{1}\right)^{2}=6^2=36$.

(c) $\left(\frac{2}{3}\right)^{-3}=\left(\frac{3}{2}\right)^{3}=\frac{3^3}{2^3}=\frac{27}{8}$.

(d) $\left(\frac{2}{3}\right)^{0}=1$.

LAWS OF EXPONENTS

Let $\frac{a}{b}$ be any rational number, and m and n be any integers. Then, we have :

(i) $\left(\frac{a}{b}\right)^{m}\times\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m+n}$

(ii) $\left(\frac{a}{b}\right)^{m}\div\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m-n}$

(iii) $\left\{\left(\frac{a}{b}\right)^{m}\right\}^{n}=\left(\frac{a}{b}\right)^{mn}$

(iv) $\left(\frac{a}{b}\times\frac{c}{d}\right)^{n}=\left(\frac{a}{b}\right)^{n}\times\left(\frac{c}{d}\right)^{n}$ and $\left\{\frac{(a/b)}{(c/d)}\right\}^{n}=\frac{(a/b)^n}{(c/d)^n}$

(v) $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$, where n is a positive integer.

(vi) $\left(\frac{a}{b}\right)^{0}=1$

Solved Examples

Example 1. *Evaluate:*

(*a*) $(-2)^{-5}$ (*b*) 5^{-3} (*c*) $\left(\frac{1}{3}\right)^{-4}$ (*d*) $\left(\frac{-3}{4}\right)^{-4}$ (*e*) $\left(\frac{5}{2}\right)^{-3}$

Solution: (a) $(-2)^{-5}=\frac{1}{(-2)^5}=\frac{1}{-2^5}=\frac{1}{-32}=\frac{-1}{32}$.

(b) $5^{-3}=\frac{1}{5^3}=\frac{1}{125}$.

(c) $\left(\frac{1}{3}\right)^{-4} = \left(\frac{3}{1}\right)^{4} = 3^4 = 81.$

(d) $\left(\frac{-3}{4}\right)^{-4} = \left(\frac{4}{-3}\right)^{4} = \left(\frac{-4}{3}\right)^{4} = \frac{(-4)^4}{3^4} = \frac{4^4}{3^4} = \frac{256}{81}.$

(e) $\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^{3} = \frac{2^3}{5^3} = \frac{8}{125}.$

Example 2. *Simplify and express with positive exponents:*

(*a*) $\left(\frac{3}{7}\right)^{5} \times \left(\frac{3}{7}\right)^{-6} \times \left(\frac{3}{7}\right)^{2}$ (*b*) $\left(\frac{5}{11}\right)^{-3} \times \left(\frac{5}{11}\right)^{-2} \times \left(\frac{5}{11}\right)^{-4}$

Solution: (a) $\left(\frac{3}{7}\right)^{5} \times \left(\frac{3}{7}\right)^{-6} \times \left(\frac{3}{7}\right)^{2} = \left(\frac{3}{7}\right)^{5+(-6)+2}$ $\left(\text{using } \left(\frac{a}{b}\right)^{m} \times \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m+n}\right)$

$= \left(\frac{3}{7}\right)^{7-6} \Rightarrow \left(\frac{3}{7}\right)^{1} = \frac{3}{7}$

(b) $\left(\frac{5}{11}\right)^{-3} \times \left(\frac{5}{11}\right)^{-2} \times \left(\frac{5}{11}\right)^{-4} = \left(\frac{5}{11}\right)^{(-3)+(-2)+(-4)}$ $\left(\text{using } \left(\frac{a}{b}\right)^{m} \times \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m+n}\right)$

$= \left(\frac{5}{11}\right)^{-9} \Rightarrow \left(\frac{11}{5}\right)^{9}$ $\left(\text{using } \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right)$

Example 3. *Simplify and express with positive exponents:*

(*a*) $\left(\frac{8}{13}\right)^{6} \div \left(\frac{8}{13}\right)^{8}$ (*b*) $\left(\frac{-3}{11}\right)^{-4} \div \left(\frac{-3}{11}\right)^{-6}$

Solution: (a) $\left(\frac{8}{13}\right)^{6} \div \left(\frac{8}{13}\right)^{8} = \left(\frac{8}{13}\right)^{6-8}$ $\left(\text{using } \left(\frac{a}{b}\right)^{m} \div \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m-n}\right)$

$= \left(\frac{8}{13}\right)^{-2} = \left(\frac{13}{8}\right)^{2}$ $\left(\text{using } \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right)$

(b) $\left(\frac{-3}{11}\right)^{-4} \div \left(\frac{-3}{11}\right)^{-6} = \left(\frac{-3}{11}\right)^{(-4)-(-6)}$ $\left(\text{using } \left(\frac{a}{b}\right)^{m} \div \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m-n}\right)$

$= \left(\frac{-3}{11}\right)^{-4+6} = \left(\frac{-3}{11}\right)^{2}$

Example 4. *Simplify the following:*

(*a*) $\left\{\left(\frac{4}{9}\right)^{5}\right\}^{-3}$ (*b*) $\left(\frac{3}{7}\right)^{-3} \times \left(\frac{14}{9}\right)^{-3}$

Solution: (a) $\left\{\left(\frac{4}{9}\right)^{5}\right\}^{-3} = \left(\frac{4}{9}\right)^{5\times(-3)}$ $\left(\text{using } \left\{\left(\frac{a}{b}\right)^{m}\right\}^{n} = \left(\frac{a}{b}\right)^{mn}\right)$

$$= \left(\frac{4}{9}\right)^{-15} = \left(\frac{9}{4}\right)^{15} \qquad \left(\text{using } \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right)$$

(b) $$\left(\frac{3}{7}\right)^{-3} \times \left(\frac{14}{9}\right)^{-3} = \left(\frac{3}{7} \times \frac{14}{9}\right)^{-3} \qquad \left(\text{using } \left(\frac{a}{b}\right)^{m} \times \left(\frac{c}{d}\right)^{m} = \left(\frac{a}{b} \times \frac{c}{d}\right)^{m}\right)$$

$$= \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} \qquad \left(\text{using } \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right)$$

Example 5. *Simplify the following:*

(*a*) $(5^{-1} \times 4^{-1})^{-1} \div 6^{-1}$ (*b*) $(6^{-1} - 4^{-1})^{-1} + (6^{-1} + 4^{-1})^{-1}$

Solution: (a) $$(5^{-1} \times 4^{-1})^{-1} \div 6^{-1} = \left(\frac{1}{5} \times \frac{1}{4}\right)^{-1} \times \frac{1}{6^{-1}} \qquad \left(\therefore a^{-1} = \frac{1}{a}\right)$$

$$= \left(\frac{1}{20}\right)^{-1} \times 6^{1} = 20^{1} \times 6^{1}$$

$$= 20 \times 6 = 120$$

(b) $$(6^{-1} - 4^{-1})^{-1} + (6^{-1} + 4^{-1})^{-1} = \left(\frac{1}{6} - \frac{1}{4}\right)^{-1} + \left(\frac{1}{6} + \frac{1}{4}\right)^{-1} = \left(\frac{2-3}{12}\right)^{-1} + \left(\frac{2+3}{12}\right)^{-1}$$

$$= \left(-\frac{1}{12}\right)^{-1} + \left(\frac{5}{12}\right)^{-1} = \left(-\frac{12}{1}\right)^{1} + \left(\frac{12}{5}\right)^{1}$$

$$= \frac{-12}{1} + \frac{12}{5} = \frac{-12 \times 5 + 12 \times 1}{5}$$

$$= \frac{-60 + 12}{5} = \frac{-48}{5}$$

Example 6. *Simplify the following:*

(*a*) $\dfrac{3^{-5} \times 10^{3} \times 5^{4}}{5^{6} \times 2^{4}}$ (*b*) $\dfrac{7^{-5} \times 2^{-5} \times x^{5} \times y^{-3}}{(14)^{-5} \times x^{-4} \times y^{2}}$

Solution: (a) $$\frac{3^{-5} \times 10^{3} \times 5^{4}}{5^{6} \times 2^{4}} = \frac{3^{-5} \times (2 \times 5)^{3} \times 5^{4}}{5^{6} \times 2^{4}}$$

$$= \frac{3^{-5} \times 2^{3} \times 5^{3} \times 5^{4}}{5^{6} \times 2^{4}}$$

$$= 3^{-5} \times 2^{3} \times 2^{-4} \times 5^{3+4} \times 5^{-6}$$

$$= 3^{-5} \times 2^{3+(-4)} \times 5^{7+(-6)}$$

$$= 3^{-5} \times 2^{-1} \times 5^{1} = \frac{5}{3^{5} \times 2}$$

(b) $$\frac{7^{-5} \times 2^{-5} \times x^{5} \times y^{-3}}{(14)^{-5} \times x^{-4} \times y^{2}} = \frac{7^{-5} \times 2^{-5} \times x^{5} \times y^{-3}}{(2 \times 7)^{-5} \times x^{-4} \times y^{2}}$$

$$= \frac{7^{-5} \times 2^{-5} \times x^{5} \times x^{4} \times y^{-3} \times y^{-2}}{2^{-5} \times 7^{-5}}$$

$$= 7^{-5} \times 7^{5} \times 2^{-5} \times 2^{5} \times x^{5+4} \times y^{-3-2}$$
$$= 7^{-5+5} \times 2^{-5+5} \times x^{9} \times y^{-5}$$
$$= 7^{0} \times 2^{0} \times \frac{x^9}{y^5}$$
$$= 1 \times 1 \times \frac{x^9}{y^5} = \frac{x^9}{y^5}$$

Example 7. *Find the value of n, if:*

(*a*) $\left(\frac{-3}{7}\right)^{5n-2} \times \left(\frac{-3}{7}\right)^{-3} = \left(\frac{-3}{7}\right)^{5}$ (*b*) $\left(\frac{2}{11}\right)^{4n} \times \left(\frac{2}{11}\right)^{2n-2} = \left(\frac{11}{2}\right)^{-6}$

Solution: (a) $\left(\frac{-3}{7}\right)^{5n-2} \times \left(\frac{-3}{7}\right)^{-3} = \left(\frac{-3}{7}\right)^{5}$

$\Rightarrow$ $\left(\frac{-3}{7}\right)^{(5n-2)+(-3)} = \left(\frac{-3}{7}\right)^{5}$

$\Rightarrow$ $\left(\frac{-3}{7}\right)^{(5n-2-3)} = \left(\frac{-3}{7}\right)^{5}$

$\Rightarrow$ $\left(\frac{-3}{7}\right)^{5n-5} = \left(\frac{-3}{7}\right)^{5}$

on comparing exponents, we get

$\Rightarrow$ $5n - 5 = 5$

$\Rightarrow$ $5n = 5 + 5$

$\Rightarrow$ $5n = 10$

$\Rightarrow$ $n = \frac{10}{5}$

$\Rightarrow$ $n = 2$

Hence, $n = 2.$

(b) $\left(\frac{2}{11}\right)^{4n} \times \left(\frac{2}{11}\right)^{2n-2} = \left(\frac{11}{2}\right)^{-6}$

$\Rightarrow$ $\left(\frac{2}{11}\right)^{4n+(2n-2)} = \left(\frac{11}{2}\right)^{-6}$

$\Rightarrow$ $\left(\frac{2}{11}\right)^{6n-2} = \left(\frac{11}{2}\right)^{-6}$

$\left(\frac{2}{11}\right)^{6n-2} = \left(\frac{2}{11}\right)^{6}$

on comparing exponents, we get

$\Rightarrow$ $6n - 2 = 6$

$\Rightarrow$ $6n = 6 + 2$

$\Rightarrow$ $6n = 8$

$$\Rightarrow \qquad n = \frac{8}{6}$$

$$\Rightarrow \qquad n = \frac{4}{3}$$

Hence, $\qquad n = \frac{4}{3}$.

Example 8. *Evaluate the following:*

(*a*) $(16807)^{\frac{-3}{5}}$ (*b*) $\left(\frac{32}{243}\right)^{\frac{-2}{5}}$

Solution: (a) $(16807)^{\frac{-3}{5}} = (7\times7\times7\times7\times7)^{\frac{-3}{5}}$

$$= (7^5)^{\frac{-3}{5}} = (7)^{-\left(5\times\frac{3}{5}\right)}$$

$$= 7^{-3} = \frac{1}{7^3} = \frac{1}{343}$$

(b) $\left(\frac{32}{243}\right)^{\frac{-2}{5}} = \left(\frac{243}{32}\right)^{\frac{2}{5}} = \left(\frac{3\times3\times3\times3\times3}{2\times2\times2\times2\times2}\right)^{\frac{2}{5}}$

$$= \left(\frac{3}{2}\right)^{5\times\frac{2}{5}} = \left(\frac{3}{2}\right)^{2} = \frac{3\times3}{2\times2} = \frac{9}{4}$$

Example 9. *By what number should* 5^{-1} *be multiplied so that the product becomes* $(-7)^{-1}$?

Solution: Let the required number be x.

According to the question,

$$(5)^{-1} \times x = (-7)^{-1}$$

$$x = (-7)^{-1} \div (5)^{-1}$$

$$x = \left(\frac{1}{-7}\right) \div \frac{1}{5}$$

$$x = \frac{-5}{7}$$

Example 10. *By what number should* $\left(\frac{2}{3}\right)^{-1}$ *be multiplied so that the product is* $\left(\frac{-3}{4}\right)^{-1}$?

Solution: Let the number be x.

Then, $\qquad \left(\frac{2}{3}\right)^{-1} \times x = \left(\frac{-3}{4}\right)^{-1}$

$$\Rightarrow \qquad \left(\frac{2^{-1}}{3^{-1}}\right) \times x = \frac{(-3)^{-1}}{(4)^{-1}}$$

$$\Rightarrow \qquad \frac{3}{2} \times x = \frac{4}{-3}$$

$$\Rightarrow \qquad x = \frac{4}{-3} \times \frac{2}{3} = \frac{8}{-9} = \frac{-8}{9}$$

Hence, the required number is $\frac{-8}{9}$.

Example 11. *By what number should* $\left(\frac{3}{7}\right)^{-3}$ *be divided so that quotient becomes* $\left(\frac{9}{49}\right)^{-2}$?

Solution: Let the required number be $\frac{p}{q}$.

Then, $$\left(\frac{3}{7}\right)^{-3} \div \frac{p}{q} = \left(\frac{9}{49}\right)^{-2}$$

$$\Rightarrow \left(\frac{3}{7}\right)^{-3} \times \frac{q}{p} = \left(\frac{9}{49}\right)^{-2}$$

$$\Rightarrow \left(\frac{7}{3}\right)^{3} \times \frac{q}{p} = \left(\frac{49}{9}\right)^{2}$$

$$\Rightarrow \frac{q}{p} = \left(\frac{7\times7}{3\times3}\right)^{2} \div \left(\frac{7}{3}\right)^{3}$$

$$\Rightarrow \frac{q}{p} = \left(\left(\frac{7}{3}\right)^{2}\right)^{2} \div \left(\frac{7}{3}\right)^{3}$$

$$\Rightarrow \frac{q}{p} = \left(\frac{7}{3}\right)^{4} \div \left(\frac{7}{3}\right)^{3}$$

$$\Rightarrow \frac{q}{p} = \left(\frac{7}{3}\right)^{4-3}$$

$$\Rightarrow \frac{q}{p} = \left(\frac{7}{3}\right)^{1} \text{ or } \frac{q}{p} = \frac{3}{7}$$

Hence, the required number is $\frac{3}{7}$.

Exercise 2A

1. Evaluate the following:

(a) $(-3)^{-4}$ (b) 4^{-3} (c) $\left(\frac{1}{2}\right)^{-5}$ (d) $\left(\frac{-2}{3}\right)^{-5}$ (e) $\left(\frac{4}{3}\right)^{-3}$

2. Evaluate the following:

(a) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2}$ (b) $\left(\frac{5}{3}\right)^{2} \times \left(\frac{5}{3}\right)^{2}$ (c) $\left(\frac{5}{6}\right)^{6} \times \left(\frac{5}{6}\right)^{-4}$ (d) $\left(\frac{9}{8}\right)^{-3} \times \left(\frac{9}{8}\right)^{2}$

3. Evaluate the following:

(a) $\left(\frac{-3}{5}\right)^{-4} \times \left(\frac{-2}{5}\right)^{2}$ (b) $\left(\frac{-2}{3}\right)^{-3} \times \left(\frac{-2}{3}\right)^{-2}$ (c) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{0}$

4. Evaluate the following:

(a) $\left\{\left(\frac{3}{2}\right)^{-2}\right\}^{2}$ (b) $\left\{\left(\frac{-2}{3}\right)^{2}\right\}^{-2}$ (c) $\left[\left\{\left(\frac{-1}{3}\right)^{2}\right\}^{-2}\right]^{-1}$

5. Evaluate $\left\{\left(\frac{4}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$.

6. Evaluate $\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-3}$.

7. Evaluate $[(5^{-1}\times 3^{-1})^{-1}\div 6^{-1}]$.

8. Find the value of:

(a) $(2^{-1}\times 3^{-1})\div 2^{-3}$ (b) $(2^{0}+3^{-1})\times 3^{2}$

(c) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

9. Simplify the following:

(a) $\frac{3^{-5}\times 2^{-6}\times 5^{4}}{6^{-4}\times 10^{2}}$ (b) $\frac{4^{-2}\times 7^{-6}\times x^{6}\times y^{-3}}{(28)^{-4}\times x^{-3}y^{6}}$

(c) $\frac{5^{-2}\times 3^{-2}\times m^{6}\times n^{-2}}{15^{-2}\times m^{-2}\times n^{4}}$ (d) $\frac{3^{-2}\times 5^{-3}\times a^{6}\times b^{4}}{6^{-2}\times 10^{-2}\times a^{-3}\times b^{-3}}$

10. Evaluate the following:

(a) $\left(\frac{81}{625}\right)^{-\frac{3}{4}}$ (b) $\left(\frac{343}{216}\right)^{-\frac{1}{3}}$ (c) $\left(\frac{169}{625}\right)^{-\frac{3}{2}}$ (d) $\left(\frac{256}{289}\right)^{-\frac{1}{2}}$

11. Find the value of x for which $\left(\frac{5}{3}\right)^{-4}\times\left(\frac{5}{3}\right)^{-5}=\left(\frac{5}{3}\right)^{3x}$.

12. Find the value of x for which $\left(\frac{4}{9}\right)^{4}\times\left(\frac{4}{9}\right)^{-7}=\left(\frac{4}{9}\right)^{2x-1}$.

13. By what number should $(-6)^{-1}$ be multiplied so that the product becomes 9^{-1}?

14. By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

15. If $5^{2x+1}\div 25=125$, find the value of x.

NUMBERS IN STANDARD FORM

A number written as $(m\times 10^{n})$ is said to be in standard form if m is a decimal number such that $1\le m\le 10$ and n is either a positive or a negative integer.

I. EXPRESSING VERY LARGE NUMBERS IN STANDARD FORM

Very large numbers can be expressed in standard form using positive exponents.

Solved Examples

Example 1. *Express each of the following numbers in standard form:*

(*a*) 5678 (*b*) 150000 (*c*) 15360000000

Solution: We may write:

(a) $5678 = 5.678 \times 1000 = (5.678 \times 10^3)$.

(b) $150000 = 15 \times 10000 = (1.5 \times 10 \times 10^4) = (1.5 \times 10^5)$.

(c) $15360000000 = 1536 \times 10000000 = (1.536 \times 1000 \times 10^7)$

$= (1.536 \times 10^3 \times 10^7) = (1.536 \times 10^{10})$.

Example 2. *The diameter of the sun is* (1.4×10^9) m *and the diameter of the earth is* (1.2756×10^7) m. *Show that the diameter of the sun is nearly* 100 *times the diameter of the earth.*

Solution: We have:

$$\frac{\text{diameter of the sun}}{\text{diameter of the earth}} = \frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^2}{1.2756} = \frac{14000}{12756} \times 100 = 100 \text{ (nearly)}.$$

$\therefore$ (diameter of the sun) = 100 × (diameter of the earth)

Example 3. *In a stack there are* 4 *books each of thickness* 24 mm *and* 6 *paper sheets each of thickness* 0.015 mm. *What is the total thickness of the stack in standard form*?

Solution: Total thickness of the stack $= (24 \times 4)$ mm + (0.015×6) mm

$= 96$ mm $+ 0.090$ mm $= (96.090)$ mm

$$= (96.09) \text{ mm} = \frac{9609}{100} \text{ mm} = \left(\frac{9.609 \times 1000}{100}\right) \text{mm}$$

$$= \left(\frac{9.609 \times 10^3}{10^2}\right) \text{mm} = (9.609 \times 10^1) \text{ mm}$$

Hence, the total thickness of the stack is (9.609×10^1) mm.

Example 4. *The distance between sun and earth is* (1.496×10^{11}) m *and the distance between earth and moon is* (3.84×10^8) m. *During solar eclipse moon comes in between earth and sun. At that time what is the distance between moon and sun*?

Solution: Required distance $= \{(1.496 \times 10^{11}) - (3.84 \times 10^8)\}$ m

$$= \left\{\left(\frac{1496 \times 10^{11}}{10^3}\right) - (3.84 \times 10^8)\right\} \text{m}$$

$= \{(1496 \times 10^8) - (3.84 \times 10^8)\}$ m

$= \{(1496 - 3.84) \times 10^8\}$ m $= (1492.16 \times 10^8)$ m

Example 5. *Write each of the following numbers in usual form:*

(*a*) 5.27×10^5 (*b*) 3.625×10^7 (*c*) 2.0001×10^8

Solution: (a) $5.27 \times 10^5 = \frac{527}{100} \times 10^5 = \frac{527 \times 10^5}{10^2} = 527 \times 10^{(5-2)}$

$= (527 \times 10^3) = (527 \times 1000) = 527000$

(b) $3.625 \times 10^7 = \frac{3625}{1000} \times 10^7 = \frac{3625 \times 10^7}{10^3}$

$= 3625 \times 10^{(7-3)} = (3625 \times 10^4)$

$= (3625 \times 10000) = 36250000$

(c) $2.0001 \times 10^8 = \frac{20001}{10000} \times 10^8 = \frac{20001 \times 10^8}{10^4} = 20001 \times 10^{(8-4)}$

$= (20001 \times 10^4) = 200010000$

II. EXPRESSING VERY SMALL NUMBERS IN STANDARD FORM

Very small numbers can be expressed in standard form using negative exponents.

Example 6. *Express each of the following numbers in standard form:*

(*a*) 0.00004 (*b*) 0.000000085 (*c*) 0.00000000376

Solution: (a) $0.00004 = \frac{4}{10^5} = (4 \times 10^{-5})$.

(b) $0.000000085 = \frac{85}{10^9} = \frac{8.5 \times 10}{10^9} = \frac{8.5}{10^8} = (8.5 \times 10^{-8})$.

(c) $0.00000000376 = \frac{376}{10^{11}} = \frac{3.76 \times 100}{10^{11}} = \frac{3.76 \times 10^2}{10^{11}}$

$= \frac{3.76}{10^{(11-2)}} = \frac{3.76}{10^9}$

$= (3.76 \times 10^{-9})$.

Example 7. *The size of a red blood cell is* 0.000007 m *and that of a plant cell is* 0.00001275 m. *Show that a red blood cell is half of plant cell in size.*

Solution: We have:

Size of a red blood cell $= 0.000007 \text{ m} = \frac{7}{10^6} \text{ m} = (7 \times 10^{-6}) \text{ m}$.

Size of a plant cell $= 0.00001275 \text{ m} = \left(\frac{1.275 \times 10^3}{10^8}\right) \text{m}$

$= \frac{1.275}{10^{(8-3)}} \text{ m} = \frac{1.275}{10^5} \text{ m}$

$= (1.275 \times 10^{-5}) \text{ m}$.

$$\frac{\text{Size of a red blood cell}}{\text{Size of a plant cell}} = \frac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \frac{7 \times 10^{(-6+5)}}{1.275} = \frac{7 \times 10^{-1}}{1.275}$$

$$= \frac{7}{(1.275 \times 10)} = \frac{7}{12.75} = \frac{7}{13} \text{ (nearly)}$$

$$= \frac{1}{2} \text{ (approximately)}.$$

$\therefore$ Size of a red blood cell $= \frac{1}{2} \times$ (Size of a plant cell).

Example 8. *Express the following numbers in usual form:*

(*a*) 5×10^{-5} (*b*) 5.21×10^{-4} (*c*) 2.687×10^{-6}

Solution: We have :

(a) $5\times10^{-5}=\frac{5}{10^5}=\frac{5}{100000}=0.00005$

(b) $5.21\times10^{-4}=\frac{521}{100}\times\frac{1}{10^4}=\frac{521}{10^2\times10^4}=\frac{521}{10^6}=\frac{521}{1000000}=0.000521$

(c) $2.687\times10^{-6}=\frac{2687\times10^{-6}}{1000}=\frac{2687}{10^3\times10^6}=\frac{2687}{10^9}=0.000002687$

Exercise 2B

1. Write each of the following numbers in standard form:

(a) 46.25 (b) 7800000 (c) 384000 (d) 279000000

(e) 5740000000000 (f) 456×10^5

2. Write each of the following numbers in usual form:

(a) 2.63×10^5 (b) 5.813×10^8 (c) 5.2364×10^7 (d) 6.8×10^4

(e) 7.39×10^6 (f) 2.789×10^9

3. The height of Mount Everest is 8848 m. Write it in standard form.

4. The speed of light is 300000000 m/sec. Express it in standard form.

5. The distance from the earth to the sun is 149600000000 m. Write it in standard form.

6. Mass of earth is (5.97×10^{24}) kg and mass of moon is (7.35×10^{22}) kg. What is the total mass of the two?

[**Hint.** *Total mass* $=[(5.97\times10^2\times10^{22})+(7.35\times10^{22})]$ kg

$=[(597\times10^{22})+(7.35\times10^{22})]$ kg

$=(597+7.35)\times10^{22}$ kg]

7. Write each of the following numbers in standard form:

(a) 0.0008 (b) 0.00000072 (c) 0.0000000423 (d) 0.0038

(e) 0.00000276 (f) 0.00000000578

8. 1 micron $=\frac{1}{1000000}$ m. Express it in standard form.

9. Size of a bacteria = 0.0000004 m. Express it in standard form.

10. Thickness of a paper = 0.03 mm. Express it in standard form.

11. Write each of the following numbers in usual form:

(a) 3.07×10^{-5} (b) 7×10^{-7} (c) 7.93×10^{-6} (d) 6.784×10^{-4}

(e) 2.9×10^{-2} (f) 3.018×10^{-3}

Exercise 2C

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. The value of $(-2)^{-5}$ is:

(a) -32 (b) $\frac{-1}{32}$ (c) 32 (d) $\frac{1}{32}$

2. The value of $\left(\frac{2}{5}\right)^{-3}$ is:

(a) $-\frac{8}{125}$ (b) $\frac{25}{4}$ (c) $\frac{125}{8}$ (d) $-\frac{2}{5}$

3. $(2^{-5} \div 2^{-2}) = ?$

(a) $\frac{1}{128}$ (b) $\frac{-1}{128}$ (c) $-\frac{1}{8}$ (d) $\frac{1}{8}$

4. $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = ?$

(a) $\frac{61}{144}$ (b) $\frac{144}{61}$ (c) 29 (d) $\frac{1}{29}$

5. $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3} = ?$

(a) $\frac{19}{64}$ (b) $\frac{27}{16}$ (c) $\frac{64}{19}$ (d) $\frac{16}{25}$

6. The value of $(3^{-1} + 4^{-1})^{-1} \div 5^{-1}$ is:

(a) $\frac{7}{10}$ (b) $\frac{60}{7}$ (c) $\frac{7}{5}$ (d) $\frac{7}{15}$

7. $\left[\left\{\left(-\frac{1}{2}\right)^{2}\right\}^{-2}\right]^{-1} = ?$

(a) $\frac{1}{16}$ (b) 16 (c) $-\frac{1}{16}$ (d) -16

8. The value of x for which $\left(\frac{7}{12}\right)^{-4} \times \left(\frac{7}{12}\right)^{3x} = \left(\frac{7}{12}\right)^{5}$, is:

(a) -1 (b) 1 (c) 2 (d) 3

9. If $(2^{3x-1} + 10) \div 7 = 6$, then x is equal to:

(a) -2 (b) 0 (c) 1 (d) 2

10. $\left(\frac{-5}{3}\right)^{-1} = ?$

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{-3}{5}$ (d) none of these

11. $\left(\frac{-3}{4}\right)^2 = ?$

(a) $\frac{-9}{16}$ (b) $\frac{9}{16}$ (c) $\frac{16}{9}$ (d) $\frac{-16}{9}$

12. $\left(\frac{2}{3}\right)^0 = ?$

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) 0

13. 0.0000463 in standard form is:

(a) 463×10^{-7} (b) 4.63×10^{-5} (c) 4.63×10^{-9} (d) 46.3×10^{-6}

14. 0.000367×10^4 in usual form is:

(a) 3.67 (b) 36.7 (c) 0.367 (d) 0.0367

15. 3670000 in standard form is:

(a) 367×10^4 (b) 36.7×10^5 (c) 3.67×10^6 (d) none of these

THINGS TO REMEMBER

- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, and m and n are integers. Then,

(i) $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$

(ii) $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$

(iii) $\left\{\left(\frac{a}{b}\right)^m\right\}^n = \left(\frac{a}{b}\right)^{mn}$

(iv) $\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$

(v) $\left\{\frac{(a/b)}{(c/d)}\right\}^n = \frac{(a/b)^n}{(c/d)^n}$

(vi) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

(vii) $\left(\frac{a}{b}\right)^0 = 1$

- **Numbers in standard form:** A number written as $(m \times 10^n)$ is said to be in standard form if m is a decimal number such that $1 \le m \le 10$ and n is either a positive or a negative integer.

ANSWER SHEET

Exercise 1A

1. (a) $\frac{-9}{15}$ (b) $\frac{15}{-25}$ (c) $\frac{-18}{30}$ (d) $\frac{27}{-45}$

2. $\frac{-4}{5}$ **3.** $\frac{-3}{7}$ **4.** $\frac{4}{-5}$

5. (a) $\frac{-2}{5}$ (b) $\frac{-1}{2}$ (c) $\frac{-3}{7}$ (d) $\frac{5}{8}$

6. (a) $\frac{-12}{5}$ (b) $\frac{-7}{10}$ (c) $\frac{7}{-12}$ (d) $\frac{-1}{3}$ (e) $\frac{-5}{8}$ (f) $\frac{-8}{7}$

7. (a) > (b) < (c) < (d) > (e) > (f) =

8. (a) $\frac{-3}{4} < \frac{-7}{16} < \frac{5}{-12} < \frac{9}{-24}$ (b) $\frac{-2}{3} < \frac{4}{-9} < \frac{-5}{12} < \frac{7}{-18}$

(c) $\frac{-9}{14} < \frac{-4}{7} < \frac{-23}{42} < \frac{13}{-28}$ (d) $\frac{-11}{15} < \frac{-7}{10} < \frac{-13}{20} < \frac{3}{-5}$

9. (a) $\frac{-7}{12} > \frac{-13}{18} > \frac{-5}{6} > \frac{23}{-24}$

(b) $\frac{-3}{10} > \frac{7}{-15} > \frac{-11}{20} > \frac{17}{-30}$

(c) $\frac{-23}{33} > \frac{-19}{22} > \frac{-39}{44} > \frac{-10}{11}$ (d) $\frac{1}{3} > -2 > \frac{-13}{6} > \frac{8}{-3}$

10. (a) False (b) True (c) True

Exercise 1 B

1. Do yourself **2.** Do yourself

3. (a) False (b) True (c) False (d) True

Exercise 1 C

1. (a) -2 (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{-10}{11}$ (e) $\frac{-6}{5}$ (f) $\frac{-3}{4}$

2. (a) $-\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $-\frac{2}{5}$ (d) $\frac{3}{20}$ (e) $\frac{1}{24}$

(f) $\frac{17}{18}$ (g) $-\frac{13}{60}$ (h) $-\frac{1}{48}$ (i) $-\frac{5}{54}$

3. Do yourself **4.** Do yourself

5. (a) $-\frac{21}{8}$ (b) $\frac{-3}{17}$ (c) 0,0 (d) $\frac{-3}{11}$ (e) $\frac{-9}{11}$ (f) $\frac{-8}{13}$

6. (a) $-\frac{1}{5}$ (b) $-\frac{21}{8}$ (c) 16 (d) $\frac{15}{7}$ (e) $\frac{19}{9}$ (f) $-\frac{24}{15}$

(g) $\frac{5}{13}$ (h) 0 (i) $\frac{29}{16}$ (j) $\frac{-38}{37}$

7. (a) $\frac{2}{7}$ (b) $\frac{13}{45}$ (c) $\frac{7}{6}$ (d) $\frac{-5}{12}$ (e) $\frac{45}{7}$ (f) $\frac{13}{9}$

(g) $\frac{29}{11}$ (h) $\frac{82}{65}$

8. $-\frac{7}{8}$ **9.** $-\frac{4}{3}$ **10.** $\frac{4}{5}$ **11.** $-\frac{1}{2}$ **12.** $\frac{12}{7}$

Exercise 1D

1. (a) $-\frac{4}{7}$ (b) 8 (c) $\frac{5}{6}$ (d) $\frac{-21}{40}$ (e) $-\frac{45}{8}$ (f) $\frac{10}{11}$ (g) $\frac{-32}{15}$

(h) $\frac{1}{8}$ (i) $\frac{5}{6}$ (j) -14 (k) 26 (l) -28

2. (a) $\frac{27}{17}$ (b) $\frac{-11}{15}$ (c) $\frac{-25}{6}$ (d) $\frac{1}{21}$ (e) $\frac{-1}{19}$ (f) $\frac{7}{2}$

(g) does not exist (h) -1 (i) $-\frac{11}{3}$ (j) $-\frac{9}{2}$

3. Do yourself **4.** Do yourself

5. (a) $-\frac{23}{17}$ (b) -38 (c) $\frac{15}{7}$ (d) $\frac{25}{-16}$

6. (a) $\frac{7}{3}$ (b) $-\frac{8}{5}$ (c) $\frac{-1}{11}$ (d) -7 **7.** Do yourself

8. (a) Multiplicative property of 0

(b) Property of multiplicative inverse

(c) Property of multiplicative identity

(d) Distributive law (e) Associative law

(f) Commutative law

Exercise 1 E

1. (a) $\frac{128}{7}$ (b) $\frac{2}{21}$ (c) $\frac{1}{16}$ (d) $-\frac{16}{15}$ (e) $-\frac{5}{2}$ (f) $\frac{32}{75}$

2. (a) False (b) False (c) False

3. (a) False (b) False (c) False (d) True

4. $\frac{4}{3}$ **5.** $\frac{3}{4}$ **6.** $\frac{-3}{16}$ **7.** $\frac{8}{3}$ **8.** $\frac{3}{4}$ **9.** $\frac{97}{33}$

10. (a) 10 (b) $\frac{4}{5}$ (c) $\frac{-14}{19}$ (d) $\frac{-3}{4}$

Exercise 1 F

1. $\frac{1}{12}$ **2.** $\frac{7}{24}$ **3.** $\frac{5}{2}$ **4.** $\frac{17}{4}, \frac{18}{4}, \frac{19}{4}$ **5.** $\frac{33}{48}, \frac{34}{48}, \frac{35}{48}$

6. $-\frac{8}{3}, -\frac{7}{3}$

7. $\frac{-9}{10}, \frac{-8}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}, \frac{-4}{10}, \frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}, 0, \frac{1}{10}, \frac{2}{10}$

8. $\frac{-8}{12}, \frac{-7}{12}, \frac{-6}{12}, \frac{-5}{12}, \frac{-4}{12}, \frac{-3}{12}, \frac{-2}{12}, \frac{-1}{12}, 0, \frac{1}{12}$

Exercise 1 G

1. $8\frac{1}{18}$ kg **2.** $5\frac{1}{10}$ m **3.** ₹ $66\frac{1}{2}$ **4.** $26\frac{5}{12}$ kg **5.** 610 m^2

6. $72\frac{1}{4}$ m^2 **7.** ₹ $2167\frac{1}{2}$ **8.** 4250 km **9.** ₹ $216\frac{3}{4}$

10. $377\frac{1}{2}$ km **11.** 400 boys **12.** $1\frac{1}{55}$ **13.** ₹ $47\frac{1}{2}$

14. $2\frac{3}{4}$ m **15.** 12 m

Exercise 1 H

1. (d) 2. (b) 3. (c) 4. (b) 5. (a)
6. (d) 7. (c) 8. (b) 9. (b) 10. (b)
11. (c) 12. (a) 13. (d) 14. (a) 15. (b)
16. (d) 17. (c) 18. (c) 19. (c) 20. (a)

Exercise 2 A

1. (a) $\frac{1}{81}$ (b) $\frac{1}{64}$ (c) 32 (d) $-\frac{243}{32}$ (e) $\frac{27}{64}$
2. (a) $\frac{243}{32}$ (b) $\frac{625}{81}$ (c) $\frac{25}{36}$ (d) $\frac{8}{9}$
3. (a) $\frac{100}{81}$ (b) $\frac{-243}{32}$ (c) 15 **4.** (a) $\frac{16}{81}$ (b) $\frac{81}{16}$ (c) $\frac{1}{81}$
5. $\frac{-4}{13}$ **6.** $\frac{19}{64}$ **7.** 90 **8.** (a) $\frac{4}{3}$ (b) 12 (c) 29
9. (a) $\frac{25}{48}$ (b) $\frac{16x^9}{49y^9}$ (c) $\frac{m^8}{n^6}$ (d) $\frac{16a^9b^7}{5}$
10. (a) $\frac{125}{27}$ (b) $\frac{6}{7}$ (c) $\frac{15625}{2197}$ (d) $\frac{17}{16}$
11. $x=-3$ **12.** $x=-1$ **13.** $\frac{-2}{3}$ **14.** $\frac{-2}{27}$ **15.** $x=2$

Exercise 2 B

1. (a) 4.625×10^1 (b) 7.8×10^6 (c) 3.84×10^5
   (d) 2.79×10^8 (e) 5.74×10^{12} (f) 4.56×10^7
2. (a) 263000 (b) 581300000 (c) 52364000
   (d) 68000 (e) 7390000 (f) 2789000000
3. (8.848×10^3) m **4.** (3×10^8) m/sec
5. (1.496×10^{11}) m **6.** (6.0435×10^{24}) kg
7. (a) 8×10^{-4} (b) 7.2×10^{-7} (c) 4.23×10^{-8}
   (d) 3.8×10^{-3} (e) 2.76×10^{-6} (f) 5.78×10^{-9}
8. (1×10^{-6}) m **9.** (4×10^{-7}) m **10.** (3×10^{-2}) mm
11. (a) 0.0000307 (b) 0.0000007 (c) 0.00000793
    (d) 0.0006784 (e) 0.029 (f) 0.003018

Exercise 2 C

1. (b) 2. (c) 3. (d) 4. (c) 5. (a)
6. (b) 7. (a) 8. (d) 9. (d) 10. (c)
11. (b) 12. (c) 13. (b) 14. (a) 15. (c)

Exercise 3 A

1. (a) 576, (b) 441, (d) 1089, (e) 4225, (f) 5625, (h) 11025 **2.** (a) 77 (b) 84 (c) 35 (d) 51
3. (a) 11,154 (b) 13,195 (c) 5,130 (d) 3,165
4. (a) 5,26 (b) 5,30 (c) 6,36 (d) 5,42 **5.** 81 **6.** 961

Exercise 3 B

1. (a) end in one zero (b) end in one zero
   (c) end in 3 zeroes (d) end in 5 zeroes
   (e) end in 2 (f) end in 3 (g) end in 7 (h) end in 8
2. (a) 324 (d) 900 **3.** (a) 961 (b) 4225 (d) 8649
4. (a) 49 (b) 144 (c) 100
5. (a) 100 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19)
   (b) 81 = (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17)
6. (a) (6,8,10) (b) (12,35,37) (c) (18,80,82)
   (d) (30,224,226)
7. (a) 149 (b) 183 (c) 75 (d) 281 (e) 435 (f) 209
8. (a) 258064 (b) 396900 (c) 96100
9. (a) 474721 (b) 793881 (c) 38416
10. (a) 9964 (b) 4899
11. (a) odd (b) even (c) odd (d) less
12. (a) T (b) F (c) F (d) F (e) F

Exercise 3 C

1. (a) 1225 (b) 2704 (c) 529 (d) 9216
2. (a) 7396 (b) 4489 (c) 65536 (d) 18769

Exercise 3 D

1. (a) 15 (b) 21 (c) 27 (d) 45 (e) 64 (f) 90 (g) 105 (h) 126
2. 7, 42 **3.** 13, 15 **4.** 900 **5.** 34 students **6.** 35 rows, 35

Exercise 3 E

1. (a) 84 (b) 95 (c) 107 (d) 119 (e) 102 (f) 134
   (g) 140 (h) 304 **2.** 12, 7569, 87 **3.** 64, 8464, 92
4. 1024, 32 **5.** 99856, 316 **6.** 3 min 16 sec

Exercise 3 F

1. (a) 1.3 (b) 5.8 (c) 8.7 (d) 12.5 (e) 0.54 (f) 1.04
   (g) 3.14 (h) 3.17 **2.** 0.95,87 **3.** 1.73 **4.** 1.73 **5.** 6.8 m

Exercise 3 G

1. (a) $\frac{11}{16}$ (b) $\frac{25}{27}$ (c) $2\frac{1}{18}$ (d) $1\frac{13}{17}$ (e) $\frac{4}{9}$ (f) $\frac{13}{17}$ (g) 126 (h) 351

Exercise 3 H

1. (c) 2. (b) 3. (c) 4. (d) 5. (c)
6. (d) 7. (c) 8. (a) 9. (d) 10. (c)
11. (b) 12. (c) 13. (b) 14. (a) 15. (c)

Exercise 4 A

1. (a) 9261 (b) 216000 (c) $\frac{343}{8}$ (d) $\frac{64}{125}$ (e) $\frac{1}{8000}$
   (f) $\frac{1}{3375}$ (g) $\frac{1000}{1331}$ (h) $\frac{2197}{1000}$
2. (b) $343=7^3$ (d) $125=5^3$ (e) $3375=(15)^3$
   (g) $9261=(21)^3$ (h) $8000=(20)^3$
3. (a) 512 (c) 1000 **4.** (a) 343 (d) 9261
5. 25 **6.** 4 **7.** 25 **8.** 7

Exercise 4 B

1. (a) 592704 (b) 314432 (c) 15625 (d) 103823